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# A Monte-Carlo game theoretic approach for Multi-Criteria Decision Making under uncertainty

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ABSTRACT

Game theory provides a useful framework for studying Multi-Criteria Decision Making problems. This paper suggests modeling Multi-Criteria Decision Making problems as strategic games and solving them using non-cooperative game theory concepts. The suggested method can be used to prescribe non-dominated solutions and also can be used as a method to predict the outcome of a decision making problem. Non-cooperative stability definitions for solving the games allow consideration of non-cooperative behaviors, often neglected by other methods which assume perfect cooperation among decision makers. To deal with the uncertainty in input variables a Monte-Carlo Game Theory (MCGT) approach is suggested which maps the stochastic problem into many deterministic strategic games. The games are solved using non-cooperative stability definitions and the results include possible effects of uncertainty in input variables on outcomes. The method can handle multi-criteria multi-decision-maker problems with uncertainty. The suggested method does not require criteria weighting, developing a compound decision objective, and accurate quantitative (cardinal) information as it simplifies the decision analysis by solving problems based on qualitative (ordinal) information, reducing the computational burden substantially. The MCGT method is applied to analyze California's Sacramento-San Joaquin Delta problem. The suggested method provides insights, identifies non-dominated alternatives, and predicts likely decision outcomes.



## 1. Introduction

Water resources management entails conflicts arising from opposing interests of stakeholders. Water conflicts arise at different levels. Conflict may occur at a local level when two farmers tap groundwater from the same aquifer or internationally when two countries have to share a water resource. The multitude of watershed management objectives almost inevitably result in conflicts among interest groups within a watershed [1]. For instance, managing a multi-purpose reservoir is always challenging because of trade-offs among objectives (e.g., hydropower, flood control, environmental, and water supply).

While conflicts over water resources existed even in ancient times, today's conflicts are more serious, mainly due to growing and more diverse demands on water resource systems. Thus, water resources planning, management, and policy-making often benefit from more than a simple single-objective cost-benefit analysis. With increasing demand and competition for water resources, Single-Criterion Decision Analysis (SCDA) is replaced with Multi-Criteria Decision Analysis (MCDA) (also often called Multi-Criteria Decision Making (MCDM), Multi-Criteria Analysis (MCA), Multi-Objective Decision Making (MODM), or Multi-Objective Analysis (MOA)). MCDA facilitates handling water resources planning and management problems [2–9] which are either: (1) Multi-Criteria Single-Decision Maker (MCSDM) problems in which a single decision maker (e.g., a reservoir operator) makes management decisions for all stakeholder groups with often incommensurable interests, or (2) Multi-Criteria Multi-Decision Makers (MCMDM) problems in which stakeholders, often with contradicting objectives, must jointly make management decisions.

Operations Research (OR) provides an appropriate framework for MCDA. A range of concepts and techniques have been proposed for handling MCDM problems within this framework, including: ELECTRE [10,11], Compromise Programming [12–14], Elimination Methods [15,16], SMART [17], TOPSIS [18], PROMETHEE [19–21], and others. Figueira et al. [22] provide a comprehensive review of the MCDA framework, concepts, methods, software, and applications. MCDA techniques have been frequently used in the literature. Wallenius et al. [23] found about 7000 publications from 1970 to 2007, using the keywords: "multiple criteria decision, multiattribute utility, multiple objective programming/optimization,

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**Acronyms**

| | |
|---|---|
| AHP | Analytic Hierarchy Process |
| DM | decision maker |
| GMR | General Metarationality |
| LMS | Limited-Move Stability |
| MCA | Multi-Criteria Analysis |
| MCDA | Multi-Criteria Decision Analysis |
| MCDM | Multi-Criteria Decision Making |
| MCGT | Monte-Carlo Game Theory |
| MCMDM | Multi-Criteria Multi-Decision Makers |
| MCSDM | Multi-Criteria Single-Decision Maker |
| MOA | Multi-Objective Analysis |
| MODM | Multi-Objective Decision Making |
| NS | Nash Stability |
| NMS | Non-Myopic Stability |
| OR | Operations Research |
| SCDA | Single-Criterion Decision Analysis |
| SEQ | Sequential Stability |
| SMR | Symmetric Metarationality |

goal programming, Analytic Hierarchy Process (AHP), evolutionary/genetic multiobjective, and vector optimization''. The water resources literature includes many applications of MCDM techniques. Wallenius et al. [23] found 267 MCDA studies in the energy and water resources areas during the past four decades. Hajkowicz and Collins [8] identified 113 water management MCA publications from 34 different countries. Cohon and Marks [24], Romero and Rehman [25], Hipel [7], and Hajkowicz and Collins [8] reviewed applications of MCDM methods for a diverse range of water resources problems.

For MCMDM problems, most MCDM techniques suggest aggregating the objectives of different decision makers and developing a compound objective to convert the multi-objective decision problem to a single-objective problem. An MCMDM problem is first transformed to an MCSDM. An all-knowing, all-powerful just decision maker then prescribes a final decision which is fair (by the stated criterion) to all interests. Thus, most MCDM techniques can be considered to be normative (prescriptive), trying to incur justice using a stated rationale. The difference in the results of broad range of normative MCDM methods reveals the ambiguity and subjectivity of ''fairness'', ''justice'', and ''equity'' in MCDA. The final solution highly depends on the selected MCDM method and often at least one decision maker would prefer another MCDM method (which improves his utility) for solving the problem, as no method can guarantee the most preferred outcome for all decision makers [3,26–28]. Existing normative MCDM methods assume perfect cooperation among the decision makers and search for non-dominated (Pareto-optimal) solutions. Thus, these methods are more suitable for MCSDM problems, where a single decision maker makes decisions, than for MCMDM problems where perfect cooperation often does not exist between the parties and the resulting outcome is not necessarily Pareto-optimal [29].

An impartial decision and selection is even more challenging when parties/criteria have unequal importance (political powers). Many weighting methods have been developed to consider the relative importance of competing criteria/decision makers [26,30–32]. Also, within each MCDA method a specific technique might exist for estimating and including such importance in a compound objective. However, the range of ''fair'' results based on different methods may become more diverse and less reliable with unequal importance, as weighting decision makers and criteria is one of the most difficult steps in MCDA and a potential source of uncertainty in the outcomes [3,33,34]. Sensitivity analysis methods [3,35–39], fuzzy decision making methods [40–56], and other approaches [39,57–60] have been proposed to address uncertainty in MCDA.

Game theory is valuable for extending classical MCDA in the water resources context. MCDM problems can be seen as games with multiple players and strategies. While most conventional MCDA methods ignore the behaviors of decision makers, which may prevent reaching the prescribed optimal solution in practice, game theory finds if optimal solutions are reachable, considering the self-optimizing attitudes of decision makers. Another advantage of game theory over conventional MCDA methods is its ability to reflect and address different engineering, socio-economic, and political characteristics of water resources problems without detailed quantitative information and without a need to express performances in conventional economic, financial, and physical terms [29,61].

This paper suggests a new method for MCDA and dealing with uncertainty in decision making. MCDM problems with finite and discrete alternatives are first examined as strategic games, where non-cooperative stability definitions (solution concepts) [61] can be applied to as a descriptive (positive) method to describe the decision makers' behavior and to predict the most likely outcome(s) or as a (normative) method for prescribing the proper outcome(s) for MCSDM or MCMDM problems. Then a Monte-Carlo Game Theory (MCGT) approach is developed for dealing with uncertainty in the performance of alternatives. The developed method is applied to a real MCMDM problem to show the applicability of the MCGT method and insights it can provide beyond conventional MCDA methods. The technique's main advantages are: (1) its flexibility as both descriptive and prescriptive method and determining the outcome(s) even when perfect cooperation is lacking between decision making parties; (2) its ability to solve the problem even in the absence of cardinal information (as required by conventional MCDA methods) as the suggested method requires only ordinal (as opposed to cardinal) performance information (which makes results more robust to performance uncertainties); and (3) the absence of the need to estimate performance in the same units, criteria/decision maker weighting, and objective aggregation.

Following presentation of the method, the next section introduces the California Sacramento-San Joaquin Delta problem as a case study. Then, the framework for studying MCDM problems as strategic games is elaborated. Later, a stochastic approach using Monte-Carlo selection is developed for making decisions when uncertainties exist about the performances of alternatives. The developed method is then applied to the California Sacramento-San Joaquin Delta problem and its results are presented and discussed.

## 2. MCDM as a strategic game

A MCSDM problem with $m$ alternatives and $n$ criteria can be defined in cardinal form as:

$$MCDM_{card} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mn} \end{bmatrix}_{m\times n} \quad (1)$$

where $P_{ij}$ is the performance of $i$th alternative under the $j$th criterion for $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$.

The same matrix can be used for a MCMDM problem with $k$ decision makers (DMs), where each DM has only one criterion, setting $n$ equal to $k$ ($n = k$). For such a problem $P_{ij}$ can be replaced with $U_{ij}$ – the utility of $j$th DM from the $i$th alternative:

$$MCDM_{card} = \begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1k} \\ U_{21} & U_{22} & \cdots & U_{2k} \\ \vdots & \vdots & \cdots & \vdots \\ U_{m1} & U_{m2} & \cdots & U_{mk} \end{bmatrix}_{m\times k} \quad (2)$$

for $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,k$.

The criteria and performances in the MCSDM problem, respectively, correspond to the DMs and utilities in the MCMDM problems in which each DM has only one criterion and vice versa. The two matrices (1) and (2) are essentially similar, so can be mathematically treated in the same way. MCDA methods can handle both types of problems in the same way [62]. Most developed methods are suitable for MCSDM problems or MCMDM problems in which each DM is summarized as only one criterion.

For a general MCMDM problem with $m$ alternatives and $k$ DMs, if each DM has $n_q$ criteria the MCDM matrix can be extended to matrix (3)

$$MCDM_{card} = \begin{bmatrix} P_{111} & P_{121} & \cdots & P_{1n1} & P_{112} & P_{122} & \cdots & P_{1n2} & \cdots & P_{11k} & P_{12k} & \cdots & P_{1nk} \\ P_{211} & P_{221} & \cdots & P_{2n1} & P_{212} & P_{222} & \cdots & P_{2n2} & \cdots & P_{21k} & P_{22k} & \cdots & P_{2nk} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ P_{m11} & P_{m21} & \cdots & P_{mn1} & P_{m11} & P_{m21} & \cdots & P_{mn1} & \cdots & P_{m1k} & P_{m2k} & \cdots & P_{mnk} \end{bmatrix}_{m\times l} \quad (3)$$

where $P_{ijq}$ is the performance of $i$th alternative under the $j$th criterion of player $q$ for $i = 1,2,\ldots,m$; $j = 1,2,\ldots,n_q$; $q = 1,2,\ldots,k$; and $l = \sum_{q=1}^{k} nq$.

Matrix (3) suggests that a MCMDM problem can essentially be solved as a MCSDM problem with $l$ criteria or a MCMDM problem with $l$ DMs where each DM only has one criterion, representing their overall preferences. Solving such a problem with existing MCDM methods is straightforward when all criteria have the same importance in matrix (3). Such a situation rarely occurs in practice as not only the DMs may have different powers, but also their criteria may not be equally important to them. Such differences make the solution based on the existing MCDM methods computationally intensive and less reliable as uncertainty increases dramatically when political importance varies across decision-makers and their criteria (matrix (3)).

To search for reliable solutions, a MCMDM problem (matrix (3)) can be modeled and studied as a strategic game, as game theory can handle MCMDM problems [29]. A strategic game (conflict) is defined as a decision situation involving more than one independent DM, who make individual choices that together determine the outcome, and who have individual preferences over the conflict's possible outcomes [63]. Here, studying the problem using non-cooperative game theory concepts is suggested. Non-cooperative game theory allows the analysis, using ordinal ranking information instead of the conventional cardinal performance or utility information [29,61]. Thus, if the DMs or a central DM (analyzer) is uncertain about the performance or utility of alternatives, the problem can be developed in an ordinal form, by replacing matrices (1) and (2) by

$$MCDM_{ord} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ R_{m1} & R_{m2} & \cdots & R_{mn} \end{bmatrix}_{m\times n} \quad (4)$$

and the general MCMDM matrix (3) by matrix (5)

$$MCDM_{ord} = \begin{bmatrix} R_{111} & R_{121} & \cdots & R_{1n1} & R_{112} & R_{122} & \cdots & R_{1n2} & \cdots & R_{11k} & R_{12k} & \cdots & R_{1nk} \\ R_{211} & R_{221} & \cdots & R_{2n1} & R_{212} & R_{222} & \cdots & R_{2n2} & \cdots & R_{21k} & R_{22k} & \cdots & R_{2nk} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ R_{m11} & R_{m21} & \cdots & R_{mn1} & R_{m11} & R_{m21} & \cdots & R_{mn1} & \cdots & R_{m1k} & R_{m2k} & \cdots & R_{mnk} \end{bmatrix}_{m\times l} \quad (5)$$

where $R_{ij}$ is the ranking of $i$th alternative with respect to the $j$th criterion and $R_{ijq}$ is the ranking of $i$th alternative with respect to the $j$th criterion of player $q$ in his view.

Soliciting the ordinal ranking of alternatives from decision-makers is less challenging than soliciting cardinal information. When analysis is based on ordinal information, the final results are less sensitive to uncertainty in the information provided by DMs, as results are insensitive to the changes in the performance or utility values as long as rankings do not change. Another benefit of studying a MCDM problem as a strategic game is the absence of need to weight DMs and criteria which also reduces uncertainty in results.

The basic elements of a MCDM problem are: (1) criteria; (2) alternatives; and (3) performances of alternatives for each criterion. These correspond, respectively, to the basic elements of a strategic game: (1) players; (2) strategies (alternatives); and (3) payoffs of players from possible outcomes, where outcomes are all possible combinations of players' strategies, as shown in Fig. 1.

Since conventional MCDM techniques only focus on group decisions, an "alternative" in the MCDM context is a possible cooperative outcome. These techniques assume a perfect cooperation among parties which allows agreement on one alternative and a cooperative outcome, disregarding the non-cooperative situations where parties may not agree, leading to non-cooperative outcomes overall. For instance, for a groundwater exploitation MCDM problem, in which two farmers want to select one of two possible pumping rates to maximize their return in a given period, conventional MCDM techniques only consider two outcomes, each occurring when both parties agree to the same alternative. Therefore, the MCDM problem is defined conventionally as a 2 by 2 matrix which provides the utility (return) of two farmers from two alternative/outcomes. Non-cooperative game theory takes a broader look at the problem by expanding the set of feasible outcomes to include both non-cooperative and cooperative outcomes. Under non-cooperative game theory, there is no pressure on the DMs to adopt cooperative alternatives/outcomes. Game theory assumes that parties are self-optimizers and may try to maximize their own benefits while considering constraints imposed on them by the decisions and actions of other DMs [29,64–66]. Thus, the structure of the problem sometimes leads to disagreement (non-cooperation) being preferred by some DMs to agreement (cooperation).

To convert a MCDM problem from its conventional form (here a 2 by 2 matrix) to a game theoretic form, a transition matrix is needed which includes both cooperative and non-cooperative outcomes. Such a matrix presents the utility of the DMs for the possible outcomes, which includes all possible combinations of each DM's alternatives. For the groundwater exploitation problem with two DMs = {Farmer 1, Farmer 2}, each with two Alternatives = {Low Pumping Rate (LPR), High Pumping Rate (HPR)}, four Outcomes = {LPR–LPR, LPR–HPR, HPR–LPR, HPR–HPR} are possible. The transition matrix for this problem in ordinal form appears in Fig. 2 (left matrix), which is a four-by-two matrix representing the payoffs of the two farmers from the four possible outcomes. Each row represents an outcome and each column represents a farmer. Therefore, the numbers in the first column indicate the payoffs of Farmer 1 from the four possible outcomes (rows of the matrix) and the numbers in the second column indicate the payoffs of Farmer 2 from the possible outcomes. This matrix corresponds to the two-by-two groundwater exploitation game [29,61], shown in normal (matrix) form in Fig. 2 (the right matrix).

To simulate the DM's behaviors in the game, predict how the game is played, and find the equilibria (possible outcomes) of games, game models apply stability definitions (commonly called *solution concepts*), which reflect different types of people with different levels of foresight, risk attitude, and knowledge of opponents' preferences [61]. Games with discrete strategies, similar to the groundwater exploitation game (Fig. 2), can be analyzed using non-cooperative stability definitions such as the Nash Stability (NS) [67,68], General Metarationality (GMR) [69], Symmetric Metarationality (SMR) [69], Sequential Stability (SEQ) [70], Non-Myopic Stability (NMS) [71], and Limited-Move Stability (LMS) [72,73], applied in various water resources conflict resolution studies [74–81]. Application of these stability definitions is suggested here to predict or prescribe the final outcome(s) of MCDM problems in a game theoretic framework. The characteristics of these stability definitions are discussed in the cited papers. Madani and Hipel [61] review specifications for each of these non-cooperative stability definitions and illustrate their utility in finding resolutions for simple water conflict problems, including groundwater exploitation (Fig. 2).

Since DM characteristics are considered in addition to their preference matrices through application of non-cooperative stability definitions, game theory can reflect behaviors of the involved parties within a MCDM process, something often neglected by conventional MCDA [29]. Hence, the results suggested by game theory for MCMDM problems are closer to practice. As discussed, MCSDM problems are mathematically the same as MCMDM problems, where each DM only has one overall criterion. However, using conventional MCDM methods (appropriate for MCSDM problems) to solve MCMDM problems neglects effects of DMs' behaviors on outcomes. Game theory results are not necessarily Pareto-optimal (non-dominated) [29], as game theory also considers DMs' possible non-cooperative behavior, which may lead to Pareto-inferior results in a group decision making context [29,61,64–66].

To find the possible results of the game, stability analysis is performed for all possible outcomes of the game. When all decision makers find an outcome stable under a given stability definition, the outcome is an equilibrium (a possible outcome of the game) under that stability definition. A state (outcome) is stable for a given player under a given stability definition if the player finds moving from that state unbeneficial. Each stability definition reflects a particular DM behavior type. Thus, to better simulate decision making with multiple DMs and increase the reliability of results, application of a range of stability definitions has been suggested, especially in absence of precise information about the behavioral characteristics of DMs [61]. An outcome which is an equilibrium under more solution concepts has a higher chance of being the final outcome of the game.

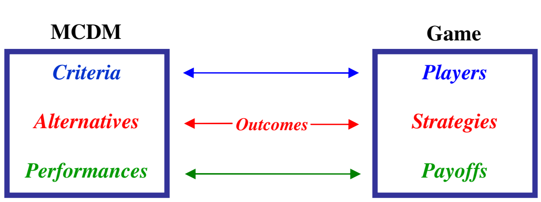


**Fig. 1.** Relationship of a MCDM and a game.

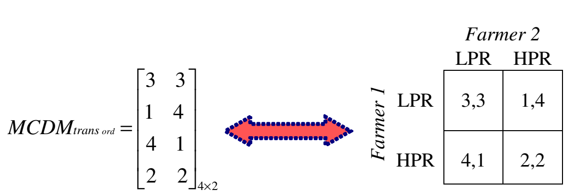


**Fig. 2.** Relationship of the groundwater exploitation game in MCDM form (left) and game theory form (right).

## 3. The Sacramento-San Joaquin Delta decision making problem

California's Sacramento-San Joaquin Delta, a part of the largest estuary on the United States' West Coast, is a vast, low-lying inland region east of the San Francisco Bay Area at the confluence of the Sacramento and San Joaquin Rivers. The Delta is home to a diverse array of fish and wildlife and is the major hub of California's water supply, providing two-thirds of the state's households and millions of acres of farmland in the Central Valley. This multi-benefit system is currently in a serious crisis. Native fish species are experiencing rapid declines, with five already listed as threatened or endangered. Delta islands face flooding due to sea level rise, land subsidence, and a weak levee system. A major earthquake may cause a catastrophic failure of the levee system, jeopardizing water supplies from the Bay Area to San Diego in Southern California. In response to declining fish populations, court rulings have cut and may continue to reduce Delta water exports. Climate change can exacerbate the crisis by changing the timing and quantity of Delta inflows, decreasing snowpack in the Sierra-Nevada mountains, and increasing sea level rise. The Delta also has lacked a strong governing institution to make critical policy decisions to reverse this deteriorating situation and satisfy stakeholders with

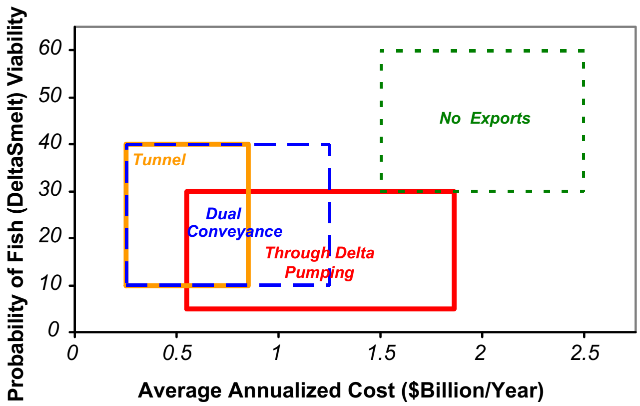


**Fig. 3.** Performance of Delta water export alternatives under two criteria (adopted from Lund et al. [85]).

strong opposing interests. The Delta system is not sustainable in its current situation, change is inevitable, and implementation of new measures and strategies is essential for retrieving the system [82,83].

Lund et al. [82] explored and compared nine available long-term strategies for solving the Delta problem, considering the environmental, economic, and water supply performances. They concluded that no single alternative is the best. However, they identified five strategies as more promising and suggested further investigation into those solutions. Lund et al. [83] continued their analysis for the Delta by analysing four strategic options for Delta water exports which are more compatible with the objectives of the Delta vision initiative. This initiative, established by Governor Arnold Schwarzenegger, established two co-equal goals for future Delta management: (1) conservation of the ecosystem; and (2) creation of a reliable water supply for California [83]. The four strategies for Delta water exports [83], are: (a) continuing to pump water through the Delta (business as usual); (b) building a tunnel (canal, tunnel, or pipeline) to move water around the Delta; (c) operating a Dual conveyance system, combining the two previous strategies; and (d) ending water exports, weaning much of California from the Delta as a water hub. Details on each alternative are given by Lund et al. [83].

Fig. 3 summarizes the performance of each alternative under the two criteria for selecting the future Delta export alternative as found by Lund et al. [83]. In their study, fish population viability was considered as the environmental sustainability criterion and the economic cost (summing implementation, maintenance, and failure costs) of each alternative as the water supply reliability criterion. The Delta has two key fish species – the delta smelt and the fall-run Chinook salmon. This study only focuses on the delta smelt which is listed as threatened under the Endangered Species Act since 1993. The analysis' results do not differ substantially if Chinook salmon is also considered. So it was decided to consider only the delta smelt in this study. This paper focuses only on determining the best water exports option out of the four suggested options of Lund et al. [83]. Solving the complex and multi-aspects Delta conflict, which has been in place for more than a century, is not the objective of this paper. Madani and Lund [84] review different aspects of the Delta problem in more detail and discuss why some parties may not be interested in developing a cooperative voluntary solution to solve the Delta crisis.

In Fig. 3, delta smelt viability is defined as sufficient recovery to have a self-sustaining population and avoid Endangered Species Act restrictions on water exports [83]. The performance of each alternative under the two criteria involves considerable uncertainty, reflected by large performance ranges in Fig. 3. The performance ranges have been estimated through an analysis which included surveys and decision tree/ spreadsheet analysis as explained in [85]. The uncertain performance makes selection of the optimal alternative more challenging. Thus, there is a need for a method which can suggest a decision or predict the final outcome of MCDM problems where the performances are uncertain. The MCGT method provides such useful insights into MCDM problems with performance uncertainty.

**Table 1**
Performance of Delta water export alternatives under two criteria [83].

| Alternative | Average annual cost ($ billion/year) | Likelihood of viable fish (delta smelt) Population (%) |
|---|---|---|
| *A* – Continuing through Delta exports | 0.55–1.86 | 5–30 |
| *B* – Tunnel | 0.25–0.85 | 10–40 |
| *C* – Dual conveyance | 0.25–1.25 | 10–40 |
| *D* – No exports | 1.50–2.50 | 30–60 |

The Delta MCDM problem can be defined as follows. This MCMDM problem can be simplified to four alternatives and two DMs – the Delta water exporters, concerned with sustainability of water exports, and environmentalists, concerned with native fish population viability. The problem also can be formulated as a MCSDM problem in which a state decision-making body would select the best Delta export alternative with respect to two criteria – the average net annualized cost and the probability of fish viability. The best alternative would have the highest fish survival and the lowest cost. However, there is a tradeoff between the two alternatives, with no alternative being the best in both aspects. To define the problem in a matrix form, performance values are required. The available performance values from Lund et al. [85] (Table 1) include an uncertainty range for each performance value and alternative. One method for dealing with situations like this is taking the averages to simplify the stochastic problem to a deterministic version of the problem. By doing so, the Delta problem can be defined in cardinal form as:

$$MCDM_{card} = \begin{bmatrix} 1.205 & 17.5 \\ 0.550 & 25.0 \\ 0.750 & 25.0 \\ 2.000 & 45.0 \end{bmatrix}_{4\times 2}$$

and in ordinal form as:

$$MCDM_{ord} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 3 & 2 \\ 1 & 3 \end{bmatrix}_{4\times 2}$$

The numbers in the left and right columns of each matrix, respectively, indicate the average utilities of the water exporters and environmentalists from the four different alternatives (average performance of the alternatives under the economic and environmental criteria) presented in Table 1 and explained earlier. In the ordinal matrix, numbers show the ranks of alternatives A–D from the corresponding DM's point of view (with respect to the corresponding column criterion). Higher ranks are preferred for the DM under that criterion. To study the Delta MCDM problem by non-cooperative game theory, a transition matrix can be developed, which includes all possible combinations of the alternatives. Since each DM has four alternatives, the transition matrix has $2^4 = 16$ rows. Developing the ranking orders in the transition matrix can be reasonably done with some judgment. In this problem, the Delta water export method is not changed from its current method (through-Delta pumping), unless both stakeholder groups agree to the new method. Thus, the outcomes in which the two DMs choose different alternatives do not allow for change from the status quo

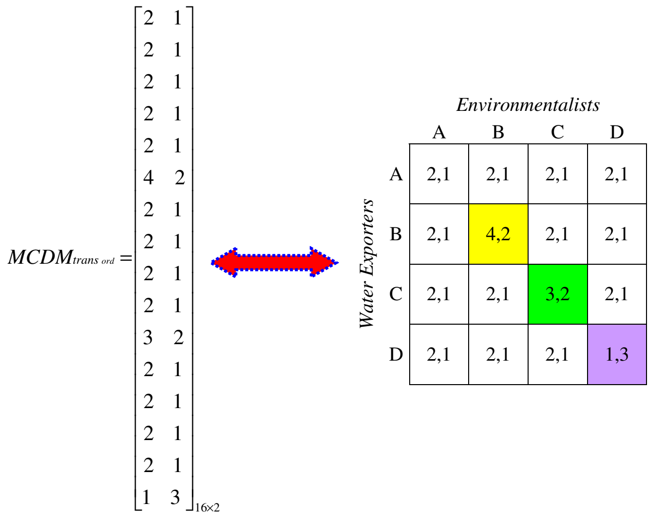


**Fig. 4.** The transition matrix for the Delta problem (left) and the corresponding game structure (right)

and through Delta exports continue (business as usual). Therefore, such disagreements (non-cooperative outcomes) have the same rank as the status quo. Based on this argument, the transition matrix and the corresponding game can be developed as shown in Fig. 4. The three highlighted cells in the right matrix show possible cooperative outcomes, occurring when both DMs choose the same strategy which differs from business as usual (A). Under all other outcomes, payoffs are the same as the status quo payoffs, as the situation cannot change from the status quo unless both parties choose the same strategy other than A at the same time. Nevertheless, parties are allowed to choose independent strategies which may not even match. With the Delta game theoretic problem expressed in a deterministic form, non-cooperative stability definitions can be applied to finding any stable outcomes and provide insights for decision making.

## 4. Deterministic game analysis results

To analyze the Delta game using the six aforementioned non-cooperative stability definitions, the GMCR II decision support system package [86,87], based on the Graph Model for Conflict Resolution [73,88] was used. Table 2 shows the stability analysis results, based on the six solution concepts, for the deterministic Delta game in which the average utilities of players (DMs) from each alternative (performances of each alternative under each criterion) were considered. The second column of the table shows the number of stability definitions which found the outcome stable for both players. For example, the status quo (continued through Delta exports) was stable under four (of six) stability definitions for both players. Thus, the status quo is an equilibrium and a likely outcome of the game, based on four different stability definitions. The "no exports" outcome was not an equilibrium under any stability definition. Therefore, ending exports is unlikely and is not suggested, as it appears to be unstable in long run, due to its high economic costs. The third column of Table 2 shows the stability strength of each outcome. An equilibrium which is stable under more stability definitions is considered to be stronger and more likely [89]. A strong equilibrium, leaves no incentive for deviation [63,89]. In this problem, choosing through Delta exports or Dual conveyance is likely and can create equilibria. However, they are not as strong and plausible as a tunnel. The parties may support the business as usual scenario or Dual conveyance. However, eventually the conflict ends only by building a tunnel.

**Table 2**
Stability analysis results for the deterministic Delta game with average utility/performance values.

| Outcome | Stable under how many stability definitions? | Stability strength | Stable under cooperation? |
|---|---|---|---|
| Continuing through Delta exports | 4 | Weak | No |
| Tunnel | 6 | Strong | Yes |
| Dual conveyance | 4 | Weak | No |
| No exports | 0 | Unstable | No |

The identified equilibria are not necessary optimal, as they are found considering the non-cooperative potential of the DMs. Similar to other MCDM methods, this game theoretic method can also prescribe non-dominated outcomes which occur under cooperation. Through cooperation, parties may be able to increase their payoffs by making agreements or changing their strategies at the same time. Cooperation does not necessarily require the agreement of all parties and can occur with an agreement between at least two parties. Coalitions (with at least two players) can be formed in which the coalition parties decide to change their strategies so they do better together, considering the threats and actions of other players. Within the suggested framework, possible cooperative outcomes are found through Coalition Analysis [63], using GMCR II. Coalition Analysis helps to identify any subsets of DMs with both opportunity and motivation to form a coalition. In other words Coalition Analysis asks which subset of DMs would gain by cooperating, and how they might coordinate their actions [63]. The problem under study has only two DMs. So, cooperative outcomes only occur when both parties are willing to cooperate. The last column of Table 2 shows which Delta game outcomes are stable under cooperation; the only stable outcome under cooperation is when both parties choose a tunnel.

In case of a single DM, as in MCSDM problems, only cooperative results should be considered, as a MCSDM problem is similar to a MCMDM problem in which cooperation among the DMs is guaranteed and the final outcomes are always non-dominated. For example, in the Delta problem, if an external DM (e.g., the state of California) is the only and final DM, construction of a tunnel is the most likely outcome of decision making. However, when the decision is made by a group, as modeled here, the non-cooperative tendencies of DMs, lack of trust among them, and absence of a clear vision and information about future makes cooperative outcomes less likely in short run. For the Delta problem, parties may prefer to hold the status quo for a time, before adopting a conveyance tunnel, either together or individually (based on self-interest). The history of the problem [84] supports this finding, as the Californians defeated a ballot initiative to build a Peripheral Canal in 1982. In recent years, building peripheral conveyance (now as a tunnel) has resurged as a Delta water export strategy. The analysis results based on average performance values show that Dual conveyance and through Delta export options are equilibria and may be experienced during the course of the game. However, they are not sustainable (strong). In the long run, construction of a tunnel remains likely even if non-cooperative behaviors remain. However, with cooperation the parties can resolve the conflict earlier by developing the strong equilibrium through mutual agreement on a tunnel.

## 5. Stochastic MCDM

So far, the Delta MCDM problem has been simplified to a deterministic problem by using averages of performance. However, using averages for ranking the alternatives can discard considerable information and make the final result less reliable, especially

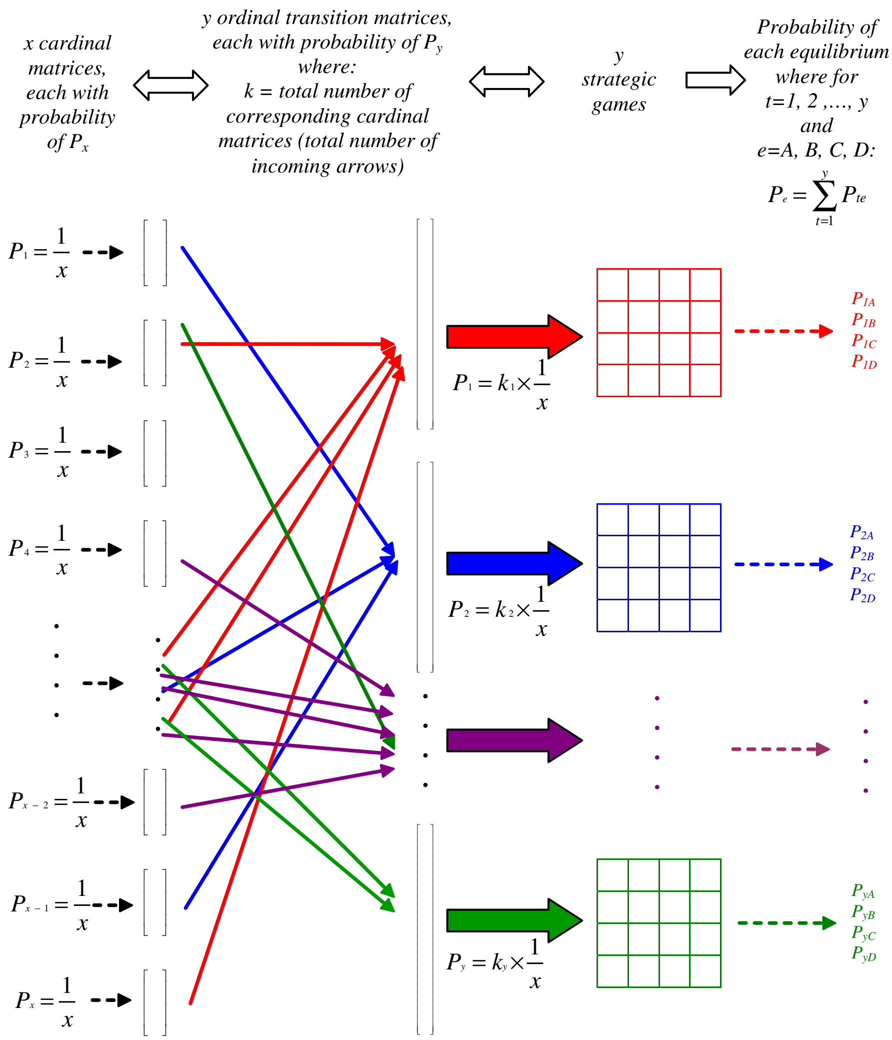


**Fig. 5.** The Monte-Carlo Game Theory (MCGT) method for solving MCDM problems with uncertainty (the Delta example with two DMs and four alternatives).

when the relations between attributes and DM's utilities are nonlinear. Uncertainty in the performance of alternatives (utility of DMs) can influence the resultant rankings. A more rigorous approach would map uncertainties in the inputs to the outputs, informing the DMs of the likelihood of suitability of different outcomes with different possible rankings of alternatives.

Using ordinal information within the suggested framework allows for a Monte-Carlo analysis without a considerable computational effort. Fig. 5 illustrates a Monte-Carlo Game Theory (MCGT) approach for solving MCDM problems under uncertainty. First, the utility of each DM from each alternative is set to a random number selected within the given performance ranges (Fig. 3 and Table 1). This random selection is repeated many (say $x$) times. Each round of random number selection generates one independent deterministic cardinal MCDM problem with probability of $1/x$. To solve the $x$ randomly generated cardinal MCDM problems, corresponding ordinal transition matrices should be developed. Since different cardinal rankings may correspond to a same ordinal ranking, $x$ cardinal MCDM matrices correspond to $y$ ordinal transition matrices where $y \leqslant x$. This reduces the computational effort significantly as instead of analyzing $x$ matrices, only $y$ matrices are analyzed. The probability of each ordinal transition matrix is $P_t = K_t/x$, where for $t = 1,2,\ldots,y$: $K_t$ is the number of corresponding cardinal matrices.

$y$ strategic games are developed based on the corresponding transition matrices. Each game is solved as a deterministic game and its equilibria are found, using the non-cooperative stability definitions (as shown earlier). The probability of each equilibrium ($e$) is equal to $P_{te}$ where $e = A,B,C,D$ in the Delta problem. After solving $y$ deterministic games and calculating the probability of each possible outcome (equilibrium) the total probability of each outcome in the overall stochastic game can be calculated as:

$$Pe = \sum_{t=1}^{y} Pte \qquad (6)$$

The suggested MCGT approach maps the stochastic MCDM problem to a deterministic environment by converting the problem into $y$ deterministic games. After solving the $y$ games, the probability of each result is calculated and stochastic results are reported. Presentation of probabilistic outcomes provides a better picture of the problem and recognizes the effects of performance (ranking) uncertainty on the final results. To explore the effects of sensitivity of the results to changes in ranking orders, instead of systematically changing one or more variables in the MCDM problem, while keeping others constant [3], the MCGT method allows for simultaneous changing of all variables of the system in different random directions and exploration of almost all possible problem structures. While such exploration may be computationally intensive using conventional quantitative/cardinal methods, the ordinal method suggested here reduces the computational effort substantially by clustering the many ($x$) generated cardinal matrices into fewer ($y$)

ordinal games while, depending on the uncertainty ranges, *y* may be many times smaller than *x*. The Delta problem can be solved using the MCGT approach to investigate how uncertainties in performance can affect the outcomes of this MCDM problem.

## 6. Monte-Carlo game analysis results

The Delta game with uncertainties was solved using the MCGT method. The first step included selecting 60,000 sets of random numbers, assuming that all points within a given range are equally probable. (Future studies, when more is known, can consider probability distributions other than the uniform distribution used here.) Thus, in each round eight numbers, representing performances of four alternatives under two criteria ($4 \times 2$), were selected within the eight given performance ranges in Table 1 and Fig. 3. In the second step, the randomly selected utilities of the two DMs from the four available strategies were ordered and clustered into an appropriate transition matrix. The 60,000 ($n$) deterministic cardinal matrices were clustered into 97 ($m$) transition matrices, making the problem 619 times smaller. In constructing the transition matrices, ranks of non-cooperative outcomes, in which players choose different alternatives, were set equal to the status quo outcome (continuation of through Delta pumping). The game structures were developed based on the transition matrices. Then each game was manually entered to the GMCR II software and solved with the six non-cooperative stability definitions. Since manual simulation of the 97 games with GMCR II was time consuming, only games with more than 0.5% occurrence chance were examined. That included 30 games, having a cumulative occurrence probability of 90.4%, leaving 67 game structures with only 9.6% cumulative occurrence probability unstudied. Automation of the last step of the MCGT approach (modeling games in GMCR II) should be considered in future studies.

Table 3 presents results from the MCGT method. Similar to the solution to the deterministic problem with average performance values, the game has three possible outcomes. The occurrence probabilities are independent as more than one outcome may be possible for a given game. The status quo is a possible outcome (equilibrium) for all modeled games. Ending water exports was never an equilibrium. The tunnel and Dual conveyance options have almost the same chance of being the final outcome of the game, both with a lower occurrence probability than existing through-Delta water exports. Third column of Table 3 shows the cumulative occurrence probability of being a strong equilibrium (most possible outcome). To be a strong equilibrium, an outcome should be stable under all six applied stability definitions. A tunnel alternative is the most broadly stable outcome (strongest equilibrium), being stable under six solution concepts more than half the time (48% out of 90%). The next strong equilibrium is Dual conveyance, having a higher chance of being stable under all solution concepts than the status quo. However, since the difference of the two probabilities is less than 10%, that finding could change if the remaining 9.5% of possible game structures are modeled. Coalition Analysis results (column 4 of Table 3) show that most of the time, under cooperation, parties are better off with a tunnel (for the prescriptive case with a single DM, e.g., state of California, only the results of coalition analysis should be considered). Although, a Dual conveyance option is weaker and less probable than a tunnel, it has a considerable chance (40% out of 62%) of being the outcome under cooperation. Although, the status quo is an equilibrium for all game structures, it is neither strong, nor stable under cooperation most of the time (76% out of 90%), suggesting that continuation of through-Delta exports is a temporary solution, and in the long run will be replaced by another alternative. Such an alternative is more likely to be a tunnel, being stronger than Dual conveyance.

It may be argued that equally preferred alternatives (which existed in the deterministic Delta game with average performances) may hardly occur when numbers are selected randomly and existence of alternatives with equal performances may change the analysis results. To address this concern and to explore its validity, a sensitivity analysis can be done by promoting equally preferred alternatives during the random selection phase, through rounding the randomly selected performance values. Here, the numbers were rounded to two decimal places. Promotion of equally preferred options increased the number of possible game structures from 97 to 375. Again, only the games with an occurrence probability exceeding 0.5% were modeled, which included 33 games with a cumulative occurrence probability of 82.11%. Table 4 shows the results of the sensitivity analysis of the stochastic Delta game, using the MCGT method. Again, the status quo was a possible outcome of all studied games, ending water exports was never a stable solution for both players. A tunnel is the most stable solution with and without cooperation, and Dual conveyance is the second most stable non-cooperative and cooperative outcome.

Since the cumulative occurrence probability spans of Tables 3 and 4 are different, Table 5 compares the results. The difference in the results is small. The only considerable difference is the strength of the tunnel construction outcome. When equally-preferred outcomes were promoted, this equilibrium was strong (stable under all six stability definitions) less frequently. Nevertheless, its chance as a strong equilibrium is double that of Dual conveyance, and it still has the highest chance of being stable under cooperation.

**Table 3**
Stability analysis results for the stochastic Delta game using the MCGT method.

| Outcome | Percent of all games[a] being | | | |
|---|---|---|---|---|
| | Equilibrium[b] (%) | Strong equilibrium[c] (%) | Stable equilibrium under cooperation (%) | Unstable equilibrium under cooperation (%) |
| Continuing through Delta exports | 90.45 | 14.17 | 14.17 | 76.28 |
| Tunnel | 64.78 | 48.00 | 57.42 | 7.35 |
| Dual conveyance | 62.20 | 18.86 | 40.53 | 21.67 |
| No exports | 0 | NA[d] | NA | NA |

[a] The analysis covers 30 games with cumulative occurrence probability of 90.445%.
[b] Probability of being a possible outcome of the game.
[c] Probability of being the highest possible outcome if the game.
[d] Not applicable.

**Table 4**
Stability analysis results for the stochastic Delta game with rounded random numbers using the MCGT method.

| Outcome | Percent of all games[a] Being | | | |
|---|---|---|---|---|
| | Equilibrium (%) | Strong equilibrium (%) | Stable equilibrium under cooperation (%) | Unstable equilibrium under cooperation (%) |
| Continuing through Delta exports | 82.11 | 12.03 | 12.03 | 70.08 |
| Tunnel | 60.48 | 33.19 | 54.01 | 6.47 |
| Dual conveyance | 57.64 | 16.08 | 36.62 | 21.02 |
| No exports | 0 | NA | NA | NA |

[a] The analysis covers 33 games with cumulative occurrence probability of 82.107%.

**Table 5**
Comparison of the results of the regular analysis (Table 3) and the sensitivity analysis with rounded random numbers (Table 4).

| Outcome | Random selection method | Percent of all modeled games being | | | |
|---|---|---|---|---|---|
| | | Equilibrium (%) | Strong equilibrium (%) | Stable equilibrium under cooperation (%) | Unstable equilibrium under cooperation (%) |
| Continuing through Delta exports | NR[a] | 100 | 16 | 16 | 84 |
| | R[b] | 100 | 15 | 15 | 85 |
| Tunnel | NR | 72 | 53 | 64 | 8 |
| | R | 74 | 40 | 66 | 8 |
| Dual conveyance | NR | 69 | 21 | 45 | 24 |
| | R | 70 | 20 | 45 | 26 |
| No exports | NR | 0 | NA | NA | NA |
| | R | 0 | NA | NA | NA |

[a] The analysis using the Not Rounded (NR) random selection covers 30 games with cumulative occurrence probability of 90.445%.
[b] The analysis using the Rounded (R) random selection covers 33 games with cumulative occurrence probability of 82.107%.

Although, building a tunnel was found to be the most promising option under the both suggested deterministic and stochastic game theory approaches, the insights provided by the stochastic approach were not obtainable through the deterministic approach. For instance, based on the deterministic method, dual conveyance and business as usual water exports methods have the same value. However, results of the stochastic method suggest that dual conveyance is superior to business as usual and sometimes it may be stable under cooperation. Furthermore, when there is uncertainty in the input performance values, a deterministic analysis which relies on average performance values provides controversial results. A stochastic analysis can increase the trust in the obtained results and reduce possible conflicts among the decision makers.

## 7. Conclusions

This study introduced an innovative approach for solving MCDM problems when there is uncertainty in performances of the alternatives. Through a Monte-Carlo simulation the stochastic MCDM problem is mapped into a deterministic environment, where many MCDM problems are generated, converted to strategic games, and solved using non-cooperative game theory concepts. The results are then mapped back to the stochastic environment and are associated with probabilities to inform the DMs of the effects of the existing uncertainty on the results.

The MCGT method can handle multi-criteria multi-decision-maker problems with discrete alternatives, which are not easily solved using other existing MCDA methods, without a need for accurate quantitative (cardinal) information, weighting of criteria and DMs, conversion of all performance values to a same unit, and developing a single compound objective. The use of ordinal ranking information allows for dealing with the stochastic aspect of the problem through a Monte-Carlo simulation, with much less computational effort. Although many games are generated through the Monte-Carlo method, they are clustered and studied in a smaller group of ordinal comparisons.

Where most MCDA methods are applicable for suggesting best solutions, the MCGT method can be used both for description and prescription. Most MCDM methods assume perfect cooperation among the DMs to find the system's optimal solutions. However, game theory also can consider cases where decision makers have self-optimizing tendencies, are unwilling to cooperate, and give priority to their own objectives [29]. As a descriptive approach, the MCGT method can describe possible changes to the game and predict the final outcomes, when the parties are willing or unwilling to cooperate. As a normative method, MCGT can be applied to prescribing stable non-dominated solutions. In the latter case, the analysis is limited to possible cooperative outcomes, where parties are willing to cooperate and improve together.

Analysis of California's Sacramento-San Joaquin Delta problem, using the MCGT approach, suggests that the existing water export method is not highly stable, and is likely to be replaced by a more stable solution. Tunnel construction was found to be the most likely alternative to replace the existing water export method. If parties are willing to cooperate, they can implement a conveyance tunnel earlier. However, if their strong opposing interests discourage cooperation, through-Delta water exports will be prolonged. The history of the conflict [84] matches this finding as conflicts have not allowed changing the Delta water export method in the past decades. Dual conveyance is the most reliable and likely solution after a conveyance tunnel. Adoption of the no water exports solution is unlikely for this problem.

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