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Voting Under Uncertainty: A Stochastic Framework for Analyzing Group Decision Making Problems

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Abstract Water resources policy making often involves consideration of a broader scope of environmental, economic, and social issues. This inevitably complicates policy making since consensus among multiple stakeholders with different interests is needed to implement decisions. This work employs several practical and popular voting methods to solve a multi-stakeholder hydro-environmental management problem. Conventionally, voting methods or social choice rules have been applied for consensus development in small groups and elections. This work combines voting methods with a Monte-Carlo selection, in order to help with social choice making under uncertainty. This process is intended to aid decision-makers with understanding of the risks associated with potential decision alternatives. The Sacramento-San Joaquin Delta's water export conflict is solved here as a benchmark problem to illustrate the proposed framework for social decision making and analysis under uncertainty.

Keywords Social choice · Voting · Monte-Carlo · Decision analysis · Group decision making · Multi-criteria decision analysis (MCDA) · Multi-criteria decision making (MCDM) · Stochastic · Uncertainty · Sacramento-San Joaquin Delta

1 Introduction

Managing water and environmental resources is a multi-faceted procedure made difficult by the need to address the objectives of every stakeholder. Over the years, hydro-environmental management problems have changed from simple cost benefit problems into more complex

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multi-criteria decision making (MCDM) problems (Mirchi et al. 2010) calling for wide application of multi-criteria decision analysis (MCDA) methods in the field (Cohon and Marks 1975; Hipel 1992; Hajkowicz and Collins 2007). MCDM problems have been divided into two different categories (Madani and Lund 2011): 1) multi-criteria single-decision maker (MC-SDM) problems; and 2) multi-criteria multi-decision maker (MC-MDM) problems. In MC-SDM problems, one decision maker serves as a social planner (Madani et al. 2013) having the responsibility of finding the best solution alternative considering the existing criteria. MC-MDM methods, on other hand, are more appropriate for water and environmental resources planning and management problems as they can effectively consider the impact of presence of multiple stakeholders on development of a stable decision which might not be necessary Pareto-optimal, as suggested by MC-SDM problems (Madani 2010; Read et al. 2014). Nevertheless, such impact has been overlooked and MC-MDM problems have been conventionally solved using MC-SDM methods by simple conversion of multiple decision makers to multiple criteria (Madani and Lund 2011).

In essence, application of conventional MC-SDM methods (Hajkowicz and Collins 2007) to MC-MDM problems implies the assumption of perfect cooperation among the decision makers. In absence of cooperation among decision makers, non-cooperative game theory concepts (Madani and Hipel 2011; Madani 2013) and bargaining methods (Brams and Kilgour 2001; Sheikhmohammady and Madani 2008; Madani et al. 2011) can be applied to find the stable outcome of the decision making process. While full cooperation is not likely to emerge when multiple stakeholders are present, parties might not pursue decision making on a fully non-cooperative basis in practice. Therefore, none of the aforementioned methods can reliably find the outcome of decision-making processes in which parties benefit from some level of cooperation.

This paper reviews a class of methods, i.e. social choice or voting methods, which are applicable to MC-MDM problems with partial cooperation. In such problems, parties normally fail to fully cooperate to achieve a Pareto-optimal outcome. Nevertheless, some level of cooperation (e.g. agreement over the option selection rule) can potentially help the parties avoid the inferior fully non-cooperative outcomes. Thus, in terms of social (system level) optimality, social choice making methods are expected to produce results that fall in between pessimistic non-cooperative game theory/bargaining approaches and optimistic MCDM approaches. Stability (acceptability or reachability) of the solutions produced by these methods increase in the reverse order, i.e. social choices are harder to implement than the pessimistic (non-cooperative) resolutions and easier to implement than the optimistic (MCDM) solutions (Read et al. 2013).

Voting is a simple, well-known, and conventional way of group decision making in small groups or even large-scale elections. Voting or social choice rules (De Borda 1781; Condorcet 1785; Dodgson 1876; Nanson 1882; Thurstone 1927; Black 1948; Arrow 1951; Brams and Fishburn 1978; Bassett and Persky 1999; Sertel and Yilmaz 1999; Nurmi 1999, 2010) are different methods of aggregation of decision makers' votes for their preferred options and have been proposed for developing transparency and fairness. Voting methods are normally simple to understand and appreciate by large group of decision makers and the general public. So, they do not cause in mistrust of the decision makers (Gregory 2002), who might find mathematically sophisticated MCDM, game theory, and bargaining methods somewhat confusing.

Voting methods do not require detailed quantitative information as they rely on ordinal information, i.e. voters' expressed preference orders over the alternatives. This makes the decision process quick, transparent, and convenient to handle even in large groups (Kangas et al. 2005). Additionally, methods that rely on ordinal ranking information are less sensitive to the uncertainty in input information when such information is provided in quantitative (cardinal) terms, making them less controversial and more robust in practice (Madani and Lund 2011).

Despite their simplicity and applicability in group decision making problems, applications of social choice rules to water and environmental management problems have been highly limited (Martin 1996; D'angelo et al. 1998; Laukkanen et al. 2002; Kant and Lee 2004; Laukkanen et al. 2004; Srdjevic 2006; Goetz et al. 2007; Sheikhmohammady and Madani 2008).

Uncertainty is integral to decision-making, but sometimes disregarded in hydro-environmental planning and management. Uncertainty could be internal—relating to the decision makers' notions and judgments (Isendahl et al. 2009; Klauer and Brown 2004), or external—relating to the imperfect information of the problem (Figueira et al. 2005). Ignoring the uncertainties involved in different components of a decision making problem, most importantly in input information, can result in misleading outcomes. A responsible and comprehensive decision making analysis must inform the stakeholders about the effects of the involved uncertainties on the selected decision and its risk of failure. Uncertainty in water and environmental decision making problems have been mainly handled through sensitivity analysis (Hyde et al. 2005), fuzzy decision analysis (Zarghami et al. 2008; Afshar et al. 2011), and Monte-Carlo selection approaches (Madani and Lund 2011). The first two methods use stochastic inputs to generate deterministic output, and thus do not fully inform the decision maker about the magnitude of risk associated with the selected outcomes. Sensitivity analysis evaluates internal uncertainty by altering performance measures and weights to test the feasibility of solutions. The fuzzy problem solving approach assumes that stakeholders' preferences have some uncertainty that can be evaluated at different degrees to evaluate its impact on the decision (Giordano et al. 2005; Rastgoftar et al. 2012). The latter method helps with mapping uncertainties from input to output while simplifying it. Therefore, instead of eliminating uncertainties from the outputs, outcomes are given in a probabilistic form, informing the stakeholders about the risks directly associated with decision analysis outputs and the outcomes they may select.

This major objective of this work is to propose a framework for social choice making under uncertainty. This framework, which combines social choice rules with a Monte-Carlo selection, is applicable to partially cooperative group decision making problems with uncertain input information on performances of alternatives under decision criteria. To illustrate how the proposed methods can be applied in practice, they are applied to analyze a real-world hydro-environmental multi-participant decision making problem –the Sacramento-San Joaquin Delta water export conflict (Lund et al. 2007, 2010; Madani and Lund 2012). In particular, this conflict is considered a benchmark problem, applied previously in the water and environmental resources management literature to illustrate different stochastic MC-MDM methods (Madani and Lund 2011; Madani et al. 2011; Mokhtari et al. 2012; Rastgoftar et al. 2012).

The paper is structured as follows: section two outlines the Sacramento-San Joaquin Delta problem; section three describes the ranking methods to evaluate the alternatives; section four solves the Delta problem with eight well established voting rules; sections five and six describe the process for handling uncertainty in the rankings through a Monte-Carlo analysis; and section seven finishes with conclusions.

2 Benchmark Problem

The Sacramento-San Joaquin Delta is located in the confluence of the Sacramento and San-Joaquin rivers at the San Francisco Bay, California. The Delta provides two-thirds of California's water and serves as home to a major ecosystem and infrastructure. Over five decades of aggressive water exports through the Delta for urban and agricultural uses, extensive developments in the Delta, and lack of strong management institution and vision have put the Delta in crisis (Lund et al. 2007, 2010). The Delta is suffering from a range of problems, calling for development of a sustainable solution that serves two equally important goals: 1) conservation of the ecosystem; and 2) providing a reliable water supply for California. Lund et al. (2010) suggested four management alternatives compatible with these objectives: 1) continuing through the Delta exports as usual; 2) constructing a tunnel to convey water around the Delta; 3) constructing a dual conveyance system for water transfers whereby water can move through the Delta's interior, and transfer in and out of the Delta depending on the demands; and 4) stopping the water exports.

The economic and environmental performance of these two options were evaluated by Lund et al. (2008, 2010), assuming that economic performance can be determined based on the cost of construction, maintenance, and failure of each project. Environmental performance is represented by the rate of fish survival under implementation of each option. Table 1 includes the performance information of the four suggested alternatives under the two equally important criteria based on experts' judgments. Performances are given in ranges, reflecting the uncertainties associated with experts' judgments.

Water exporters and environmentalists are the two major groups of stakeholders in the Delta problem (Madani and Lund 2012). Environmentalists evaluate the alternatives based on their environmental performance, i.e. they prefer alternatives with higher fish survival rates, and the water exporters evaluate the alternatives based on their economic performance, i.e. they prefer alternatives with lower costs associated with water exports. Given the existing trade-offs and the uncertainties involved (Table 1), these parties have not been able to develop a compromise solution to date.

Using the stochastic group decision making problem of Delta as a benchmark this work illustrates how Monte-Carlo social choice making can help with solving partially cooperative group decision making problems under uncertainty. Here, the partial cooperation assumption implies that while parties cannot agree to the Pareto-optimal outcome, which results from social-planner (MC-SDM) methods, they can agree over the procedure (voting in this case) for selecting the best alternative.

For illustrative purposes we first simplify the stochastic Delta decision making problem to a simple deterministic problem by using performance range averages as performance values. The deterministic Delta decision making problem can be represented by the cardinal matrix given below. Each number in this matrix indicates the average performance of the row alternative under the column criterion.

		Cost	Fish Survival
Cardinal performance matrix =	*Through Delta Exports*	1.205	17.5
	Tunnel	0.550	25.0
	Dual Conveyance	0.750	25.0
	Stop Exports	2.000	45.0

Table 1 Economic and environmental performances of Delta water exports alternatives (Lund et al. 2008, 2010)

Alternative	Cost (billion $/year)	Fish survival (%)
Through delta exports	0.55–1.86	5–30
Tunnel	0.25–0.85	10–40
Dual conveyance	0.25–1.25	10–40
Stop exports	1.25–2.5	30–60

Voting rules determine the best alternative based on ordinal information (ranking). Therefore, the cardinal decision making matrix must be converted to an ordinal one before applying voting rules. In cases of strict dominance of performances, rank determination is straightforward; however, when equal performances exist (fish survival for tunnel and dual conveyance, e.g.), ranking orders are affected by the ordinal ranking method.

3 Ranking Methods

As indicated in the cardinal performance matrix, the fish survival rate under the tunnel and dual conveyance alternatives is equal. In this case, selecting a certain ranking strategy may bias the final outcome in the voting problem since each ranking method deals with the cases involving equally good alternatives differently. Here, four potential ranking strategies are discussed and implemented to explore the potential influences of ranking method on the solution of the benchmark problem.

3.1 Standard Competition Ranking

This type of ranking is normally applied in competitions. If competitors perform the same, their rank numbers will be equal and the next best competitor receives a rank number with a gap after the equally acting ones. The size of the gap is equal to the number of equally ranked competitors. For example, if two alternatives tie for a criterion, the rank number of the next best alternative is equal to two plus the rank of the equally ranked alternatives. Based on this method, the rank for continuing through delta exports under fish survival criterion is 2+2=4. The matrix below displays the deterministic Delta decision making problem under standard competition ranking with lower numbers indicating a better performance (higher ranking).

		Cost	Fish Survival
Standard competition ranking =	*Through Delta Exports*	3	4
	Tunnel	1	2
	Dual Conveyance	2	2
	Stop Exports	4	1

3.2 Modified Competition Ranking

If the gap in the competition ranking stays before a set of equally performing alternatives, the ranking strategy becomes a modified competition ranking. Based on this method, the ranks for the tunnel and dual conveyance are both three. The modified matrix below shows the modified competition rankings of the alternatives.

		Cost	Fish Survival
Modified competition ranking =	*Through Delta Exports*	3	4
	Tunnel	1	3
	Dual Conveyance	2	3
	Stop Exports	4	1

3.3 Dense Ranking

This method ranks alternatives with the same value equally and does not add a gap between the better and worse alternatives. The matrix below shows dense rankings of the alternatives.

		Cost	Fish Survival
Dense ranking =	*Through Delta Exports*	3	3
	Tunnel	1	2
	Dual Conveyance	2	2
	Stop Exports	4	1

3.4 Ordinal (Strict Order) Ranking

This method gives a distinct rank to every alternative, even ones that perform equally under the same criterion. Receiving a higher rank for equal alternatives is decided by chance or is arbitrary. The following matrices show the two possible representation of the deterministic Delta problem based on ordinal ranking.

		Cost	Fish Survival
Ordinal ranking 1 =	*Through Delta Exports*	3	4
	Tunnel	1	3
	Dual Conveyance	2	2
	Stop Exports	4	1

		Cost	Fish Survival
Ordinal ranking 2 =	*Through Delta Exports*	3	4
	Tunnel	1	2
	Dual Conveyance	2	3
	Stop Exports	4	1

Given that the resulting ranks are different based on the tested ranking methods, we solve the benchmark problem based on all possible rankings. In practice, decision makers can agree over the ranking method before election.

4 Voting (Social Choice) Rules

Decision making based on social choice rules requires voters to state their preferences for each alternative, where it is assumed that parties have equal powers. Each social choice rule finds the socially optimal choice by aggregating voters' preferences based on its specific notion of social optimality and fairness. Here, some popular and practical voting rules are introduced and applied to the Delta problem to demonstrate how social choice making under uncertainty can be implemented in practice and what type of insights could be expected.

4.1 Plurality

Plurality rule is one of the oldest and perhaps the most commonly used social choice making methods (Madani and Dinar 2012). This method does not need complete preference information and can determine the winning option (social choice) knowing only the most preferred option of each voter. Based on the plurality rule, the social choice solution is the alternative with the highest number of votes in favor. Looking at the ordinal matrices, the tunnel is the most preferred option for water exporters, and the stop exports option is the best for the environmentalists. Since both options receive one vote (and there are only two voters), both options can be declared as winners based on the plurality concept. In the deterministic Delta problem, this rule essentially suggests that parties will not agree on the best option, however, can agree that continuing Delta export and building a dual conveyance system are not socially optimal.

4.2 Borda (De Borda 1781)

The Borda rule scores alternatives based on their ranks. Based on this rule, every alternative receives a score according to its rank for each voter (under every criterion). When a lower rank means a better option, if the number of alternatives equals j and i represents the rank of the alternative for one voter (criterion), the Borda score of this alternative for that voter will be j-i. After scoring options for each criterion, the total Borda score (count) of an alternative is obtained by summing the scores under each criterion. The alternative with the highest Borda count is the winner based on the Borda rule.

Table 2 indicates the Borda scores of the Delta problem's solution alternatives based on the standard competition ranking. There are four alternatives in the problem ($j=4$). So, the Borda score of each alternative under a given criterion is equal to 4 minus its rank under that criterion. For example, the rank for the tunnel alternative is 1($i=1$) for water exporters, so its cost score is $4-1=3$. Rank of this option is 2 ($i=2$) for the environmentalists, so its fish survival score is 4-2=2. The Borda score for the tunnel is therefore $3+2=5$ which is higher than the Borda scores of all other alternatives. Thus, tunnel is the socially optimal option based on the Borda rule.

Applying the Borda rule to the Delta problem under modified competition ranking, dense ranking matrices, and ordinal ranking 2 results in selecting the tunnel as the socially optimal solution. In case of ordinal ranking 1, both the tunnel and dual conveyance options are Borda winners.

4.3 Condorcet (Condorcet 1785)

Pairwise comparison of alternatives under different criteria is the basis of the Condorcet method. A strong Condorcet winner is an alternative which is able to overcome all other rivals in all pairwise comparisons.

Table 2 Borda scores under the standard competition ranking

Decision Options Criteria	Cost rank	Cost score	Fish survival rank	Fish survival score	Borda score (count)
Through delta exports	3	4-3=1	4	4-4=0	1
Tunnel	1	4-1=3	2	4-2=2	5
Dual conveyance	2	4-2=2	2	4-2=2	4
Stop exports	4	4-4=0	1	4-1=3	3

Table 3 illustrates the results of applying the Condorcet method to the standard competition ranking matrix. In the left section of the table, the left number in each comparison cell shows the number of voters that preferred the row alternative to the column alternative, and the right number of the comparison cell is number of voters that prefer the column alternative to the row alternative. The right side of the table shows the number of wins, losses, and ties for each alternative. In the Delta problem a strong Condorcet option requires three wins. Table 3 shows that the tunnel beats dual conveyance and continuing through delta exports, but equals the stop exports option. This is because water exporters' best alternative is the tunnel, whereas the best environmental solution is to stop exports. Since there is a tie in the competition, a strong Condorcet winner does not exist in this case.

Because of the position of stop exports as the best alternative for the environmentalists and the worst one for the water exporters, this problem cannot have a strong Condorcet winner under all ranking methods.

4.4 Pairwise comparison (Thurstone 1927)

Pairwise comparison method is a less restricted version of strong Condorcet rule. The socially optimal option based on this method is winner of pairwise comparisons by majority (based on the plurality rule). Alternatives receive a score of 1 for winning and 0.5 for a tie in every pairwise comparison. The candidate with the highest score is the pairwise comparison or Condorcet (not strong Condorcet) winner.

Based on Table 3, the total scores of continuing through delta exports, tunnel, dual conveyance, and stop water export are 0.5, 2.5, 1.5, and 1.5, respectively. Therefore, tunnel is the pairwise comparison or Condorcet winner of the Delta problem under the standard competition ranking. The outcome is the same under modified competition ranking, dense ranking matrices, and ordinal ranking 2. In case of ordinal ranking 1, both tunnel and dual conveyance are pairwise comparison (Condorcet) winners.

4.5 Approval Vote (Brams and Fishburn 1978)

Approval voting is a less restricted form of plurality rule based on which voters are allowed to vote for more than one alternative. Similar to the plurality rule, ranking of options is not required. Essentially, voters reveal their approved options by saying yes or no to each option. The winner is then determined based on the plurality rule, i.e., the alternative with the most number of votes wins.

In the Delta problem, we have no information about the number of options that each party aims to vote for. Nevertheless, parties are not expected to vote for their worst option. Thus, here we assume that parties may select between 1 to 3 options and solve the problem for all possible combinations of voters' 1–3 selections. Table 4 represents all the possible conditions that may occur in Delta's solution selection, allowing parties to select between 1 to 3 methods,

Table 3 Pairwise comparison of alternatives under standard competition ranking

Alternatives	Through delta exports	Tunnel	Dual conveyance	Stop export	Win	Loss	Tie
Through delta exports	–	0<2	0<2	1=1	0	2	1
Tunnel	2>0	–	2>1	1=1	2	0	1
Dual conveyance	2>0	1<2	–	1=1	1	1	1
Stop exports	1=1	1=1	1=1	–	0	0	3

based on standard competition ranking. The last column of the table indicates the socially optimal choice based on approval voting. For example, if water exporters are allowed to select one option only, tunnel is their choice (second column of the table) and if the environmentalists

Table 4 Approval voting analysis based on standard competition ranking

Voters				Election Outcome
Environmentalists		Water exporters		
Selected candidates	Number of selections	Selected candidates	Number of selections	
Stop exports	1	Tunnel	1	Tunnel Stop exports
Stop exports Tunnel	2			Tunnel
Stop exports Dual conveyance	2			Tunnel Stop exports Dual conveyance
Stop exports Tunnel Dual conveyance	3			Tunnel
Stop exports	1	Tunnel Dual conveyance	2	Tunnel Stop exports Dual conveyance
Stop exports Tunnel	2			Tunnel Dual conveyance
Stop exports Dual conveyance	2			Dual conveyance
Stop exports Tunnel Dual conveyance	3			Tunnel Dual conveyance
Stop exports	1	Tunnel Dual conveyance Through Delta exports	3	Tunnel Stop exports Dual conveyance Through Delta exports
Stop exports Tunnel	2			Tunnel
Stop exports Dual conveyance	2			Dual conveyance
Stop exports Tunnel Dual conveyance	3			Tunnel Dual conveyance

are allowed to select two options, the stop exports and tunnel are their selected options. The socially optimal solution for this combination is the tunnel based on the approval voting rule. This option is the winner in 10 of the 12 possible combinations, and is more likely to win approval voting than any other option. Components of Table 4 do not change with other ranking methods.

4.6 Median Voting (Black 1948; Bassett and Persky 1999)

Based on the median vote rule, alternatives that receive at least the majority of support in the highest possible level are considered as the socially optimal solution. Ties are allowed under the median voting scheme.

Table 5 indicates the quality of support for the four alternatives of the Delta problem under standard competition ranking. This table shows that the tunnel and stop exports options have only one supporter at the 1st level, i.e. only one supporter ranks this option as the most favorite. At this level, no option wins by majority. Tunnel and dual conveyance have two supporters at the 2nd level, meaning that there are two supporters who rank these options at rank second or better. These two options win majority at the 2nd level, so they are both socially optimal solutions based on median voting. In this case, 2nd level is the highest level at which an alternative can win a majority.

Median voting based on the modified competition ranking, dense ranking and ordinal ranking 2 also select the tunnel and dual conveyance as social choices. In case of ordinal ranking 1, dual conveyance is the social choice solution.

4.7 Majoritarian Compromise (Sertel and Yilmaz 1999)

Majoritarian compromise is a refinement of the median voting scheme and the majoritarian compromise winners are a subset of median voting winners (Sheikhmohammady and Madani 2008). This rule follows the same logic as median voting, but pays a special attention to quality of support (number of supporters) besides majority. The majoritarian compromise winner must receive both the maximum and majority of votes at the highest level.

Based on Table 5, both the tunnel and dual conveyance options are the socially optimal choices based on majoritarian compromise under standard competition ranking as they receive the majority of votes (2) and the highest number of votes (2) at the highest possible level (ranked as second). So, the majoritarian compromise and median voting winners are the same in this problem, and this is true for all ranking methods.

4.8 Condorcet's Practical Rule

The Condorcet practical's method is a modified version of the Condorcet method, which has also been proposed by Condorcet according to Nanson (1882) and Nurmi (1999). Based on this scheme, the process of selecting the social choice starts with searching for the majority of

Table 5 Number of supporters at different ranks under standard competition ranking

Water export alternatives	1st	2nd	3rd	4th
Through delta exports	0	0	1	2
Tunnel	1	2	2	2
Dual conveyance	0	2	2	2
Stop exports	1	1	1	2

support at the highest level (ranked first). If there is no majority at the first level, an alternative which receives the highest level of support (though not necessarily the majority) at the second level is the Condorcet's practical winner.

For the Delta decision making problem represented by Table 5, no alternative gains majority of support at the first level. At the second level both tunnel and dual conveyance options have two supporters (total number of votes at the first and second rank). Given that two is the maximum number of votes an alternative has received at the second level, both options are selected as social choices based on the Condorcet's practical rule.

Results under dense ranking are the same as the results under standard competition ranking. The tunnel, dual conveyance and stop exports alternatives are the Condorcet's practical winners under modified competition ranking. Dual conveyance is the ordinal 1 selection and tunnel is the ordinal 2 selection under the Condorcet's practical rule.

Table 6 summarizes the results of the social choice analysis of the deterministic Delta decision making problem under different voting and ranking methods. Based on the obtained results the tunnel is the most robust social choice solution, which is optimal under most voting and ranking methods. The dual conveyance alternative is the second best option.

5 Social Choice Making under Uncertainty

The previous section solved the Delta group decision making problem deterministically by using performance averages. While this may be an effective screening method, taking averages as representatives of ranges will cause to omit some useful input information and can be the source of uncertainty itself (Madani and Lund 2011). Thus, the need to develop a better approach for characterizing the uncertainty in the group decision making problem still remains. Following Madani and Lund (2011) a Monte-Carlo selection can be used to generate a large number of performance values within given performance ranges and producing all possible combinations of performance values, i.e. all possible group decision making problem structures. This will help map the stochastic Delta problem into many deterministic ones, which will be solved using social choice methods.

In the Delta problem, eight numbers can be randomly selected out of the eight performance ranges of the problem. For illustrative purposes and following other stochastic MC-MDM studies of the benchmark problem (Madani and Lund 2011; Madani et al. 2011; Mokhtari et al. 2012; Rastgoftar et al. 2012), the probability distribution of performance values are considered to be uniform within each performance range. This means that the probability of selecting each number in the range is equal. In each round of selection, eight generated numbers constitute a cardinal performance matrix (like the cardinal performance matrix in Section 2) that will be then converted to an ordinal ranking matrix based on a desired ranking method. Thus in each round, a deterministic problem is created and solved using a social choice rule. The winning probability of each option is updated in each round of selection by simply dividing the number of times it has been selected as a winner to the number of Monte-Carlo selections. The option with the highest election probability is considered to be the most robust socially optimal solution of the partially cooperative group decision making problem.

The benchmark problem was solved in this study, using the suggested Monte-Carlo social choice making approach. In each round of selection, the problem was solved using the eight social choice rules reviewed in Section 4. Given that the randomly generated numbers were not rounded (four decimal places here) the chances of having equal performances were minimal. When equal performances do not exist, all ranking methods result in the same rank orders. Therefore, one ranking method (standard competition ranking) was used here to develop the

Table 6 Social choices of the deterministic Delta decision making problem for different voting and ranking methods

Social choice rule	Socially optimal alternative				
	Standard competition	Modified competition	Dense	Ordinal 1	Ordinal 2
Plurality	Tunnel, stop exports	Tunnel, stop exports	Tunnel, stop exports	Tunnel, stop exports	Tunnel, stop exports
Borda score	Tunnel	Tunnel	Tunnel	Tunnel, dual conveyance	Tunnel
Condorcet choice*	Tunnel	Tunnel	Tunnel	Tunnel	Tunnel
Pairwise comparison	Tunnel	Tunnel	Tunnel	Tunnel, dual conveyance	Tunnel
Approval voting	Tunnel	Tunnel	Tunnel	Tunnel	Tunnel
Median voting	Tunnel, dual conveyance	Tunnel, dual conveyance	Tunnel, dual conveyance	Dual conveyance	Tunnel, dual conveyance
Majoritarian compromise	Tunnel, dual conveyance	Tunnel, dual conveyance	Tunnel, dual conveyance	Dual conveyance	Tunnel, dual conveyance
Condorcet's practical method	Tunnel, dual conveyance	Tunnel, dual conveyance, stop exports	Tunnel, dual conveyance	Dual conveyance	Tunnel

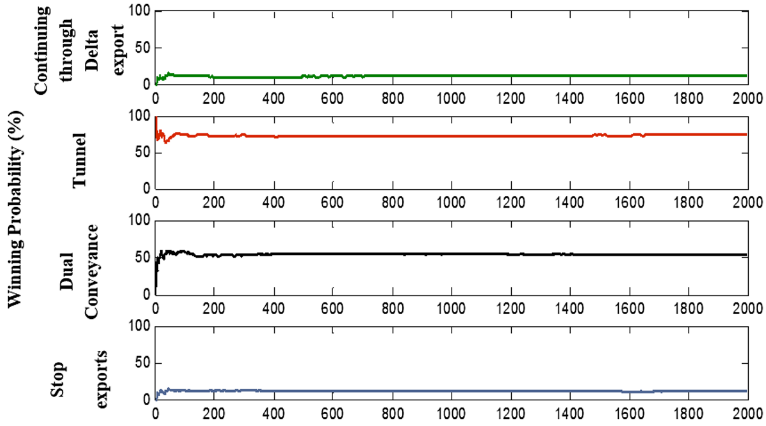

Fig. 1 Winning probability changes during the Monte-Carlo social choice making process based on Borda score

ordinal ranking matrix in each round of selection. The winning probabilities were updated at each round of selection to determine the sufficient number of Monte-Carlo selections to get stable results. As an example, Fig. 1 shows how the winning probabilities change during the Monte-Carlo social choice making procedure using the Borda score. As shown in this figure, winning probabilities converge after few hundred rounds of Monte-Carlo selection. The convergence trend was the same for other social choice rules, so the number of Monte-Carlo selections was set to 2000 for this analysis. Figure 2 shows the winning probabilities of the Delta problem's alternatives based on the suggested Monte-Carlo social choice making method. Given the possibility of ties, the winning probabilities might exceed 100 % under each voting method (in each row of the table). The obtained results suggest that the tunnel is the most robust socially optimal solution, followed by dual conveyance. The general findings are consistent with findings of other stochastic MC-

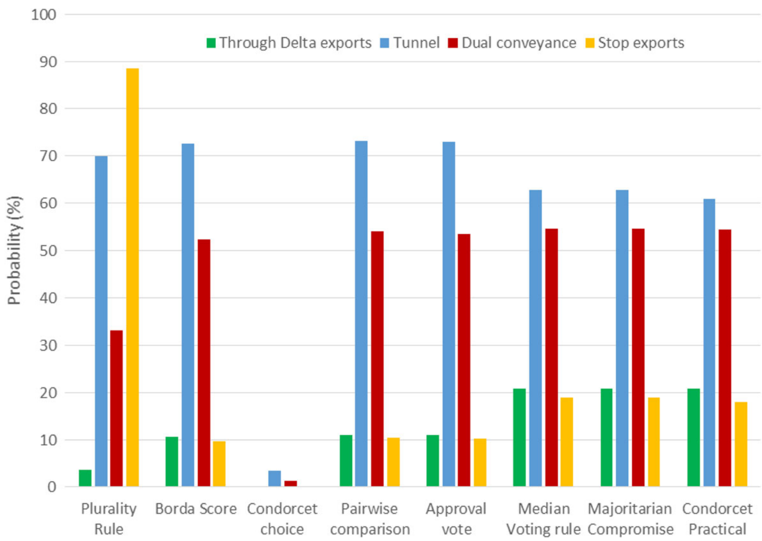

Fig. 2 Summary of winning probabilities based on different Monte-Carlo social choice methods

MDM studies of the benchmark problem, reflecting the reliability of the proposed method. Nevertheless, given the different assumptions of each study (e.g. cooperation level, fairness/optimality notion) the calculated winning probabilities are different from the other studies as expected.

The obtained results are based on unrounded selected performances, making the results insensitive to the ranking methods. To evaluate the sensitivity of the winning probabilities to possibility of having equal performances and to different ranking methods, the Monte-Carlo social choice making procedure was repeated using rounded (with two decimals) performances. Considering the possibility of equal performances with rounded performances and given the sensitivity of social choice rules to different ranking methods (as shown in Section 3) the Monte-Carlo social choice making analysis was performed with rounded performances based on all ranking methods illustrated earlier. In case of the ordinal ranking method which ranks equally performing options randomly (as shown by the ordinal ranking 1 and 2

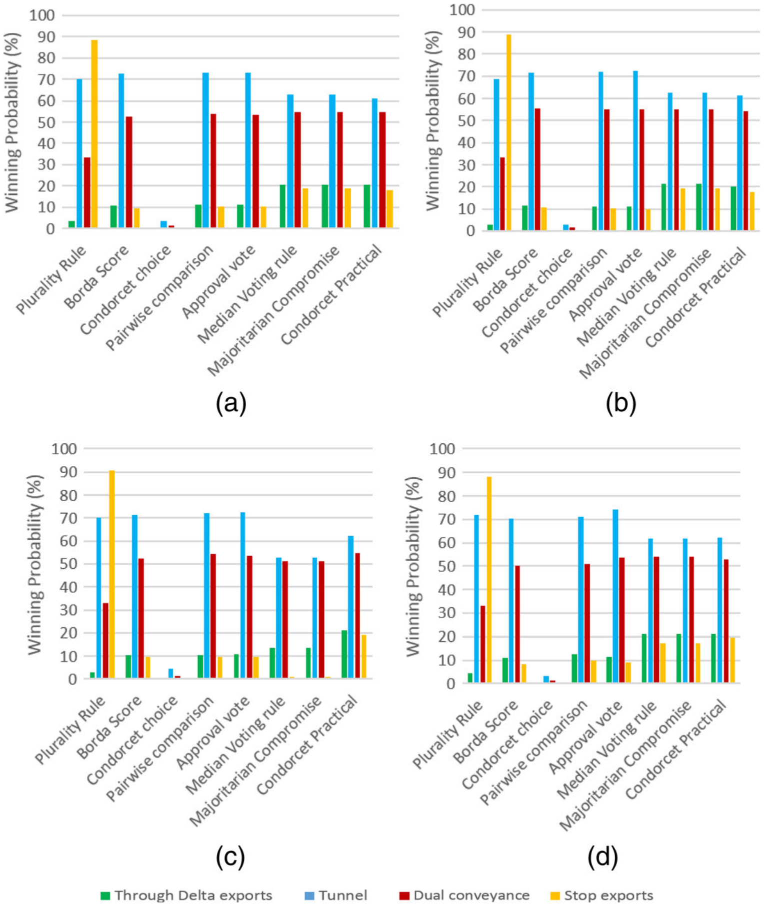

Fig. 3 Winning probabilities based on different Monte-Carlo social choice making rules for each alternative (through delta exports, tunnel, dual conveyance, and stop exports) with rounded performances based on each ranking method: **a** *SCR* standard competition ranking; **b** *MCR* modified competition ranking; **c** *DR* dense ranking; **d** *OR* Ordinal ranking

matrices), a random ranking function was added for ranking equally performing alternatives, giving them equal chances of selection at each rank.

Figure 3a–d presents the results of the Monte-Carlo social choice analysis of the Delta decision making problem with rounded numbers. Slight changes of the winning probabilities under different ranking methods indicate the sensitivity of social choice rules to ranking methods. Nevertheless, these changes are not significant and do not change the overall results, i.e. the tunnel and dual conveyance remain as the best and second best social choices, respectively. Although insignificant, the difference in winning probabilities in Fig. 3a–d reflect the sensitivity of results to equal performances. Again, the possibility of equal performances does not change the overall results.

6 Ranking Distributions

Winning probabilities help identifying the alternatives' likelihood of getting selected as the social choice, but it does not provide any information on the degree of robustness of winning options. To evaluate the robustness of an alternative, its ranking distribution can be calculated. Social choice ranking of alternatives in a deterministic group decision making problem can be established by eliminating the social choice under a given social choice rule from the decision making problem and applying that rule again to the remaining alternatives to find the next best social choice. This procedure can be continued until all choices are ranked. Ranking distributions of the alternatives in a Monte-Carlo social choice making procedure can be determined if alternatives are fully ranked in each round of selection. Ranking distribution of an alternative reflects the probabilities of that alternative being ranked at different levels and reflects the degree of robustness of each rank.

The alternatives of the Delta problem were ranked using the Monte-Carlo social choice making procedure with unrounded performance values. Figure 2 shows the ranking distributions of the four Delta problem's solution alternatives under different social choice rules. Figure 3a–d clearly show that the sum of the probabilities of selection at a given rank might exceed 100 % because of the possibility of ties. Nevertheless, for a given alternative, the sum of selection probabilities at different ranks always equals 100 %. Given that Condorcet rule cannot determine the best choice under most cases in the Monte-Carlo social making process, ranking distributions could not be established under this rule.

The ranking distributions imply the ranking robustness of each alternative. For example, tunnel as the best option is less robust under median voting, majoritarian compromise, and Condorcet's practical rules than plurality, Borda, pairwise comparison, and approval voting rules. This is because under plurality, the tunnel can rank as the third option, when it is not selected as the best option (Fig. 2). This type of information can be shared with decision makers to inform them about the risk of selecting each option in stochastic decision making problems. As another example, while winning probabilities of stop exports and continue through delta exports (1st rank) are close under Condorcet's practical method, the former is a less robust option due to its low ranking probability at the 2nd level. Thus, a conservative decision maker might prefer the continue through delta exports option over the stop exports option. Overall, Fig. 2 suggests that tunnel is the best and relatively robust social choice, followed by dual conveyance. Interestingly, the probability of getting ranked at the 4th level is minimal for all options (Fig. 4a–f), suggesting that the overall performances of the other two alternatives, stop exports and continue through delta

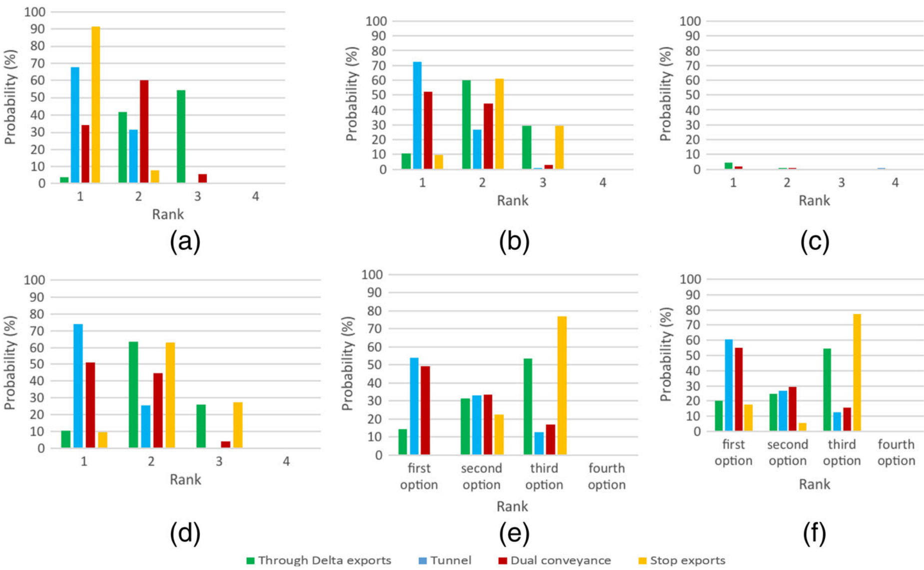

Fig. 4 Ranking distribution of alternatives under different social choice rules (**a** plurality, **b** Borda and pairwise comparison, **c** Condorcet, **d** approval voting, **e** median voting and majoritarian compromise, and **f** Condorcet's practical)

exports, are nearly the same. Ranking distributions the alternatives in the Delta problem provides decision makers with a way to easily distinguish performance while also considering risk and trade-offs.

Given that the results of the previous section showed that rounded performances do not change the overall results, significantly, ranking distributions were not determined for the rounded performances case. Nevertheless, ranking distributions are expected to be sensitive to the choice of ranking method.

7 Conclusions

This paper developed a new framework for analyzing partially cooperative group decision making under uncertainty. In this type of problem parties who have uncertain information about the performances of different alternatives, benefit from some level of cooperation in the decision making process (e.g. agreement over the decision making process and rules). The proposed Monte-Carlo social choice making method combines Monte-Carlo selection with a range of social choice (voting) rules to determine the winning probability of each alternative and the degree of ranking robustness (ranking distribution). Given that the social optimality notions of voting rules are different application of different methods help with finding a robust social ranking of the alternatives. The proposed method can be practical in decision making under uncertainty. The objective of the developed framework is not to provide a deterministic solution to a stochastic problem. Rather, it tries to inform the decision makers about the effects of uncertainty on robustness of the analysis outcomes and about the risk of selection of each alternative.

The suggested method was applied to a real-world hydro-environmental group decision making example as a benchmark problem to illustrate how the suggested method work and to show what type of insights could be expected from this method. The general findings of the study are consistent with findings of other studies of the benchmark problem, reflecting the reliability of the proposed method. Under partial cooperation the tunnel is the best option for the Delta problem, followed by dual conveyance. For illustrative purposes, this work used a relatively simple benchmark problem and assumed uncertainty to be uniformly distributed. Future studies may apply the suggested Monte-Carlo social choice making method to more complex problems and/or assume non-uniform probability distributions.

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