7th Meeting on Game Theory and Practice Dedicated to Energy, Environment and Natural Resources
May 28-30, 2007, Montréal, Canada

Improving Game Models by Considering Different Solution Concepts: A Case of an Interconnected Hydro-Conflict

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Abstract

Game models are blamed for not being able to predict the final resolutions of real conflicts in which people's behavior might be different from what the game model assumes. However, the associated assumptions of the applied solution concepts in the model should not be neglected when interpreting the results. If players play the game in a way that a particular solution concept assumes, the resolution of the game in practice will not be different from what the model predicts. The main reason for model's failures in prediction of the real resolution is the modelers' failure in applying proper solution concepts to solve the game. To better reflect the human behavior in the decision making process, different solution concepts have been proposed, trying to better simulate behaviors of different types of people with different levels of foresight, risk attitude, and knowledge of others' preferences. Such solution concepts can be applied to more accurately find the most likely resolutions of a game. In this study, some solution concepts will be applied to predict the possible resolutions of an interconnected water game between two countries. To show the value, applicability, and reliability of these solution concepts, results will be compared with results of another study which has used a different approach to find the possible resolutions of the same game.

Keywords: Game Theory, conflict resolution, solution concept, water resources

Introduction

Models are not perfect. Modeling is always associated with simplifications which result in inaccuracies. Thus, those simplifications should be considered when interpreting the results. Often game models' predictions are different from reality. This might be due to modelers' failure in using proper solution concepts for solving the game. In order to find the possible resolutions of the game, the modeler defines solution concepts and solves the game based on the assumptions associated with those solution concepts. If enough attention is not paid to define appropriate solution concepts for the game studied, reasonable results should not be expected out of the model. This is like simulating a reservoir operation when the modeler defines operational rules different from practice. In

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such a situation, the modeler cannot blame the simulation model if its results are different from what observed. Game models are nothing but simulation models. If wrong rules are defined, wrong results will be obtained.

Defining an appropriate solution concept for a particular game is not an easy task, especially if the game under study is not a historic case. There are always uncertainties associated with games, currently being played or the games which will be played later on. Assuming that the modeler has identified all the players of the game, their options and their preferences over the possible outcomes, some uncertainties will still remain about the players' behaviors in the game. People are different, so their behaviors are not the same. The modeler should search for the best solution concept for the game under study. He/she can pick an already defined solution concept or can define a new solution concept as long as it reflects the game's reality. However, this is not easy. It is like trying to find a proper probability distribution function for a small sample of data to make some estimations of interest. An example of this is in flood management studies. When there is at most 100 year of runoff data available, hydrologists struggle to find the proper probability distribution in order to estimate how big the largest flood can be in a 10,000 years period. Several probability distributions might fit the sample data and it is hard to say which one is the best one. In such situations, it might be safer to use different probability distributions and accept some kind of average of the results as a solution to the problem. The same thing can be done while modeling games. Instead of only applying Nash solution concept, the game can be solved using different solution concepts. If a particular state turns out to be equilibrium under different solution concepts, it has more chance to be the final resolution of the game.

Here, some previously used solution concepts will be introduced. Later on, such solution concepts are applied to resolve an international game of aquifer sharing under unequal access (Just and Netanyahu, 2004), which allows comparison of the results and finding the reliability of the introduced solution concepts. The results show how implementing more solution concepts into the game model makes it more accurate.

Solution Concepts

The elements of the game are players (decision makers), options (strategies or moves available to those players), and their preferences over the possible outcomes (states). Outcomes or states of a game are different combinations of strategies. A player's preference orders over a set of outcomes can be cardinal when the player knows her payoff over the outcomes or ordinal when the player can only rank the outcomes but does not know the exact value of payoffs over possible outcomes of the game.

Stability analysis is central to game analysis. A stability definition or solution concept used to identify an expected resolution of the conflict model is a description of human behavior under the assumption of rationality, as stipulated in rational choice theory. Solution concepts reflect different styles of behavior incorporating a players' level of foresight, willingness to make strategic concessions, risk attitude, and knowledge of others' preferences (Obeidi et al., 2002). Game models are often blamed for being unable to predict the outcome of the real conflicts because what happens in practice is different from the model results. In fact, game models are nothing but abstract representation of a conflict in terms of mathematics. To find the equilibria, the models, follow the rules of logic and the solution concepts incorporated in the model. Modeling is always associated with simplification, but if a model is too simplified, it neglects some facts necessary to be considered while searching for solutions. Many facts, such as the trust between the players, their risk tolerances and foresights, etc. might be missed by the modeler while constructing the game model, resulting in the failure in finding the satisfactory outcomes.

Some of the facts associated with conflicts might be hard to be modeled. However, the solution concepts can always be improved to better reflect the reality in decision making process. It might be possible to strengthen the game analysis by considering more solution concepts while analyzing the game. A given equilibrium is stronger if it is stable under more solution concepts. This means, such a state is stable for different decision makers with different risk tolerances and behavior.

Nash Solution Concept (Nash 1950, 1951) is the most familiar solution concept in the area of non-cooperative Game Theory. There are solution concepts other than Nash stability (Nash 1950, 1951) such as General Metarationality (GMR) (Howard, 1971), Symmetric Metarationality (SMR) (Howard, 1971), Sequential Stability (SEQ) (Fraser and Hipel, 1979), Non-Myopic Stability (Brams and Wittman, 1981), and Limited-Move Stability (Kilgour et al., 1987) which have been applied only by few people in the water area. These solution concepts have shown to be reliable in predicting the final resolution of different historic water and environmental resources conflicts (Fang et al., 1993; Obeidi et al., 2002; Noakes et al, 2003; Hamouda et al., 2005) and if introduced to water academics, it can open a whole new window for them to look at water conflicts, especially, in games where social and political issues are also involved.

Kilgour et al. (1984) compared mathematically a wide range of solution concepts. Table 1 outlines some of the solution concepts mentioned above. This table shows how different solution concepts can represent different decision makers with different characteristics. A player with high foresight thinks further ahead. Nash Stability has low foresight, and the level of the foresight increases from Nash Stability with the lowest foresight to Non-Myopic Stability with the highest foresight (the left column of the table). Limited-Move Stability has variable foresight level given by the number of movements considered by the player before making her decision. Some solution concepts, such as Limited-Move and Non-Myopic stabilities allow strategic disimprovements, which occur when a player temporarily moves to a worse state in order to reach a more preferred state eventually. Other solution concepts, such as Nash Stability and SEQ, never allow disimprovements. Others, such as GMR and SMR permit strategic disimprovements by opponents only. Different solution concepts also imply different levels of preference knowledge. Under Nash Stability, GMR and SMR a player needs to only know her own preferences, while for SEQ, Limited move and Non-Myopic stabilities the player must know the preference information for all other players of the game. (Hipel el al., 2003)

Table 1-Solution concepts and human behavior (Obeidi et al., 2002)

Solution Concept	**Characteristics**				**Stability Description**
	Foresight	***Disimprovement***	***Knowledge of Preferences***	***Strategic Risk***	
Nash Stability	Low (1 move)	Never	Own	Ignore	Decision maker cannot unilaterally move to a more preferred state
General Meta-Rationality (GMR)	Medium (2 moves)	By Opponent	Own	Avoid	All of the player's unilateral *improvements* are sanctioned by subsequent unilateral *moves* by others
Symmetric Meta-Rationality (SMR)	Medium (3 moves)	By Opponents	Own	Avoid	All of the player's unilateral *improvements* are still sanctioned even after possible responses by the original player
Sequential Stability (SEQ)	Medium (2 moves)	Never	Own	Takes some risks	All of the player's unilateral *improvements* are sanctioned by subsequent unilateral *improvements* by others
Limited-Move Stability	Variable number of moves	Strategic	All	Accepts	All players are assumed to act optimally and maximum number of state transitions is specified
Non-Myopic	Unlimited number of moves	Strategic	All	Accepts	Limiting case of limited move stability as the maximum number of state transitions increase to infinity

Here, Nash Stability, GMR, SMR, and SEQ will be introduced and then are applied to a hydro-conflict example:

1) *Nash Stability (Nash, 1951):* A state *k* is Nash-Stable for player *i*, iff the set of player *i*'s unilateral improvements from state *k* is an empty set ($S_i^+(k)=\varnothing$). In other words, if player *i* cannot do nay better by changing her decision, given the decisions of her opponents, she has no incentive to move from state *k*. Therefore, state *k* is Nash-Stable for her. If state *k* is Nash-Stable for all players, *k* is a Nash Equilibrium (there is no player, who can do any better by changing her option, given the options of her opponents).

Figure 1 shows the game Prisoner's Dilemma (PD) in a Normal Form. Each cell contains two values. The first value (left) represents player 1's payoff while the second value represents player 2's payoff. The strategies which yield the payoffs of each cell are given in left of the table for the first player and on top of the table for the second player. The higher the payoff, the better is the outcome. The given payoffs are ordinal. This means that the outcome (C, C) which occurs when both players decide to confess, is better for player 1 than the state (DC, C) in which she does not confess but player 2 confesses (2>1). However this does not necessarily mean that the utility of player 1 at state (DC, C) is two times greater than her utility at state (C, C) ($2 \neq 2\times1$).

		Player 2	
		DC	*C*
Player 1	*Don't Confess (DC)*	3,3	1,4
	Confess (C)	4,1	2,2

Figure 1- Prisoner's Dilemma in Normal Form

State (C, C) is the only pure strategy Nash-Equilibrium of the game since it is the only outcome which is Nash-Stable for both players. State (DC, DC) is not Nash-Stable for any of the players. State (DC, C) is Nash-Stable for the player 2 but not Nash-Stable for player 1. The opposite is true for state (C, DC). Therefore, if players behave rationally as Nash Solution Concept suggests they end up in state (C, C) while the Pareto-Optimal state is (DC, DC). A Nash-Player has a very low foresight, does not know anything about her opponent's preferences and takes no risk. Not all players in the real world are Nash-Players. In practice, depending on the conditions of the game, players might decide to play the game differently to end up in the optimal result (i.e. state (DC, DC)). Such players are not Nash-Players. To model their behaviors, other solution concepts might be considered.

2) General Meta-Rationality (GMR) (Howard, 1971): State k is GMR-Stable for player i iff unilateral improvement of player i from k can be sanctioned by player j's movement. Note that in response to player i's improvement from k to q, player j may even hurt herself by moving to state z with a lower payoff for both players to sanction player i's improvement. Therefore payoff of state z can either be higher or lower than state q for player j but it is definitely lower for player i. In such a situation, player i prefers not to move from k. Therefore, k is GMR-Stable for player i. If a given state is GMR-Stable for all players of the game, that state is a GMR-equilibrium. In the PD game (Figure 1), state (DC, DC) is GMR-Stable for both players. Thus, (DC, DC) is a GMR-equilibrium.

GMR Solution Concept simulates the behavior of a very conservative player who is aware of her opponents' preferences over the possible states. Such a player avoids any risk in making decisions. It should be noted that GMR is only applicable to repeated games with at least two moves available to each player since in one-shut games, there is no counteraction available to player *j* in respond to player *i*'s action. A GMR player has a horizon two moves away while a Nash player has only one move.

3) Symmetric Meta-Rationality (SMR) (Howard, 1971): State *k* is SMR-Stable for player *i* iff not only unilateral improvement of player *i* from *k* to *q* is sanctioned by player *j*'s movement from *q* to *z*, but also there is no unilateral movement available to player *i* from *z* to *y* where payoff of player *i* at *y* is higher than her payoff at *k*. SMR is a more restrictive stability definition than GMR and in fact a subset of GMR. SMR is like GMR except that player *i* considers not only her own possible moves and possible reactions of player *j* to that move, but also her chances to respond to player *j* reactions. SMR player has a horizon three moves distant and she anticipates that the conflict ends after her counterresponce. An SMR player is a very conservative player with a better foresight than a GMR player. This player assumes that opponents might even hurt themselves in order to sanction her moves.

In the PD game state (DC, DC) is SMR-Stable for Player 1. If she moves her strategy from DC to C, player 2 responds by changing her strategy also from DC to C, as she is better off in (C, C) relative to (C, DC). In this situation, player 1 can only react by switching from C to DC which does not make her better off. Therefore, it is better for player 1 not to change her decision and stays at (DC, DC). Similarly, (DC, DC) is stable for player 2. Thus, state (DC, DC) is an SMR-Equilibrium.

4) Sequential Stability (SEQ) (Fraser and Hipel, 1979): SEQ is a subset of GMR and a restricted version of GMR in which player *j* can only respond to player *i*'s unilateral improvement by a credible action (a unilateral improvement not a unilateral movement). This means that a state *k* is SEQ for player *i* iff she deterred from taking any unilateral improvement from *k* because of a credible action by *j* which results in a state less preferred (for player *i)* than *k.* State (DC, DC) in the PD game (Figure 1) is sequentially stable for both players and thus, it is an equilibrium.

An *SEQ* player has a medium foresight (a horizon two moves distant) and is not as conservative as SMR and GMR players as she takes some risks by assuming that her opponents are never willing to hurt themselves in order to sanction her unilateral improvements.

Figure 2 indicates the interrelationships of the solution concepts, introduced so far. Nash Stability with its limited foresight and number of moves it considers is the subset of SMR, SEQ and GMR. SMR and SEQ are both subsets of GMR. Therefore, a state (C, C) in the PD game (Figure 1) which is a Nash Equilibrium is also an equilibrium under SMR, SEQ and GMR.

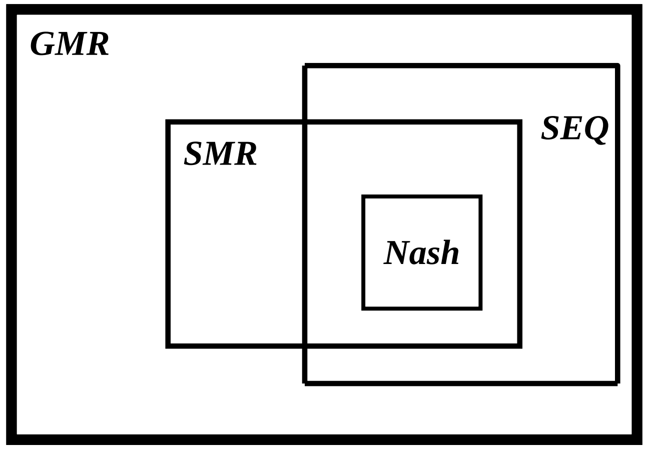

Figure 2- Interrelationships of solution concepts (Li et al., 2004)

Game of Aquifer Sharing under Unequal Access

Just and Netanyahu (2004) presented an example of hydro-conflict over sharing of groundwater between two countries. The conflict is over a common pool aquifer. Access to the aquifer is unequal because of differences in elevations of the two countries which result in a deep water table and high pumping costs for Country B. On the other hand, much of water percolates rapidly downhill to two major springs in Country A, making extraction cost negligible for this Country. Having no other water sources, Country B highly depends on sharing water with Country A, while the political relation of the two parties is not good. A solution for the problem might be that Country B bribes the country A to be able to have more water. (Just and Netanyahu, 2004)

Figure 3 shows the water-sharing problem in the Normal Form. Here, the payoff values are ordinal. Strategies of Country A are Water Sharing (WS) and No Water Sharing (NWR) while the other country's strategies are Payment (P) and No Payment (NP). The status quo of the problem is (NWS, NP). Just and Netanyahu (2004) argued that victim-pays might be infeasible in international context and in situations like this, it might be possible to link the game to another game in which Country B has an advantage over Country A. In this way, side payments can be avoided and because of a credible threat by the other country, both parties may choose a cooperative strategy. They suggest linking a control-of-smuggling game to the water sharing game. Country B has an advantage relative to Country A in enforcing laws against illegal agricultural trade and has the power of controlling illegal trade of Country B's products into Country A. This control will increase the national welfare of Country A. Figure 4 shows this game in a normal form. Again, ordinal payoffs are presented. (NP, NC) is the status quo of the game. This game also has the structure of PD game and non-cooperative strategy is a dominant strategy for each party. In absence of good political relation between the two parties and because of the problem being international, side-payments might not be feasible. (Just and Netanyahu, 2004)

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		Country B Payment (P)	No Payment (NP)
Country A	*Water Sharing (WS)*	3,3	1,4
	No Water Sharing (NWS)	4,1	2,2

Figure 3- The water sharing game in Normal Form (Just and Netanyahu, 2004)

		Country B Control (C)	No Control (NC)
Country A	*Payment (P)*	3,3	1,4
	No Payment (NP)	4,1	2,2

Figure 4- The control-of-smuggling game in Normal Form
(Just and Netanyahu, 2004)

Linking the two games result in a larger game, shown in Figure 5. Just and Netanyahu (2004) explained how they came up with a game with this structure by summing up the cardinal payoffs from all strategies in the two isolated games. In this study, only ordinal payoffs are presented as the value of payoffs do not matter in the final resolution of the game as long as the structure of the game has not changed. Each country has four strategies in the interconnected game.

		Country B *P C*	*P NC*	*NP C*	*NP NC*
Country A	*WS P*	7,9	3,10	4,12	1,13
	WS NP	10,7	5,8	7,10	3,11
	NWS P	10,3	5,4	6,6	2,8
	NWS NP	11,1	8,2	9,4	4,5

Figure 5- The interconnected game in Normal Form (Just and Netanyahu, 2004)

Solution

Here, the four solution concepts introduced earlier (Nash, GMR, SMR, and SEQ) are applied to find the possible resolutions (only pure strategy equilibria) of the games discussed so far. As was mentioned earlier, the water sharing game is a PD game in which (NWS, NP) is an equilibrium based on Nash, GMR, SMR, and SEQ solution concepts. This is one of the possible resolutions of the game as (NWS, NP) is stable under all the solution concepts considered here. Another possible resolution is the state (WS, P) which is stable under all the solution concepts considered, except the Nash Solution Concept. This is the Pareto-Optimal resolution of the game which might be neglected if Nash Solution Concept is applied for prediction of the final resolutions or equilibria are found based on iteration of the dominated strategies. Similarly, (NP, NC) and (P, C) are two possible outcomes of the control-of-smuggling game. These are the results which game models suggest, based on the solution concepts applied. However, considering the fact that victim-pays are infeasible, (WS, P) and (P, C) cannot be the final outcomes of the individual games. The water sharing and control-of-smuggling games also have mixed strategy equilibria. However, for the same reason (infeasibility of victim-pays), those equilibria cannot be the possible outcomes of the game. This is when interconnection of the games which expand the set of feasible outcomes can be beneficial as Just and Netanyahu (2004) suggested.

Infeasibility of side-payments makes the strategy sets of the players smaller. Therefore, shaded cells of Figure 6 should be omitted for they are associated with at least one strategy which includes side-payment. This results in a smaller game, shown in Figure 7. Ordinal payoffs used in the revised game to show how each player ranks the feasible outcomes. This game has structure of the PD Game. Therefore, outcomes (NWS, NC) and (WS, C) are two possible resolutions of the game based on the solution concepts considered in this analysis (Nash, GMR, SMR, SEQ) as discussed earlier. Pareto-Optimal outcome of the game is (WS, C) which is stable under GMR, SMR, and SEQ.

Using a different approach, Just and Netanyahu (2004) found (WS, C) as an optimal outcome and suggested it as a possible outcome of the game. However, they did not found this based on a particular solution concept. They reasoned that the parties do not like to defer from their cooperative strategies (WS for Country A and C for Country B) because of the credible threat available to the other party. This means that if country A decides to change the outcome from (WS, C) to (NWS, C) to increase its payoff, Country B responds by changing the outcome to (NWS, NC). Since (NWS, NC) is worse than (WS, C) for Country A, it will never switch from its cooperative strategy to its non-cooperative strategy. This is exactly the behavior which solution concepts such as GMR, SMR, and SEQ simulate. This shows how by defining proper solution concepts for a given conflict, game models can generate satisfactory results. When there are uncertainties about the behavior of the players, incorporating different solution concepts into the model is beneficial.

		Country B			
		P C	P NC	NP C	NP NC
Country A	WS P	7,9	3,10	4,12	1,13
	WS NP	10,7	5,8	7,10	3,11
	NWS P	10,3	5,4	6,6	2,8
	NWS NP	11,1	8,2	9,4	4,5

Figure 6- The interconnected game in Normal Form (shaded cells are the infeasible outcomes)

		Country B	
		C	NC
Country A	WS	3,3	1,4
	NWS	4,1	2,2

Figure7- Revised interconnected game in Normal Form

Just and Netanyahu (2004) only discussed the infeasibility of mixed strategy equilibria which are associated with side-payments in the studied interconnected game. However, they did not make any comment about the mixed strategy equilibrium of the refined game (Figure 7) without any side-payment. This game has a mixed strategy equilibrium. However, mixed strategy equlibria may not be acceptable in an international context either because of the credible threat available to the involved parties or because of treaties and agreements among the countries. In situations like this, countries prefer to stick to a pure strategy rather than playing mixed strategies which results in loss of trust between the conflicting parties.

Conclusions

Conflict modeling is always associated with simplifications and inaccuracies resulting from the limited information about the game. Although it is impossible to come up with a perfect model which is able to predict the real outcomes of the conflicts, it is possible to reduce the model errors in outcome prediction by incorporating more solution concepts into the model. Solution concepts reflect the behavior of the players while making a decision. Such behavior depends on different factors considered by the player, her risk attitude, and the information available to her while playing the game. Selection of a proper solution concept is a challenging task. However, it is possible to solve the game using different solution concepts. A state which is stable under different solution concepts has a higher chance of being the final resolution of the game, if reachable from the status quo.

Here, a pre-studied game was solved using solution concepts other than Nash Stability Definition. Results show how considering more solution concepts can improve the accuracy of predictions.

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